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New properties of the attenuated V-line transform for breast cancer detection with Compton cameras

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***Abstract*—In the recent past, Compton camera became an attractive alternative to the Anger camera in scintimammography, known as nuclear medicine breast imaging or molecular breast imaging. This novel imaging modality leads to the use of the V-line transform, which integrates a function along coupled rays with a common vertex. In previous works the attenuation phenomena was mostly neglected. However, in scintimammography ignoring the effect of the attenuation of photon can significantly degrade the quality of the reconstruction image. In this paper, we introduce the attenuated V-line transform and establish a new integral relation between the attenuated V-line transform and the exponential Radon transform. The results are not only interesting as original mathematical discoveries, but also can be useful in challenging applications e.g., in breast imaging for tumor detection close to the chest wall.**

***Keywords*-V-line transform; attenuated V-line transform; imaging of breast cancer; breast cancer detection; Compton camera.**

I. Introduction

According to the American Cancer Society, in the United States, breast cancer is one of the most commonly diagnosed cancer among women and it has the second highest mortality rate [1]. Statistic shows that one in eight women will have breast cancer in her lifetime [1]. Despite the high chance of breast cancer related death, early diagnosis and timely treatment can decrease the fatality rate [2].

Breast cancer screening has made significant advancement in recent years. Currently, X-ray mammography and ultrasonography are the two main techniques applied for breast cancer detection. According to Lee et al., the widespread use of mammographic screening has led to nearly 30% decrease in breast cancer mortality since 1990

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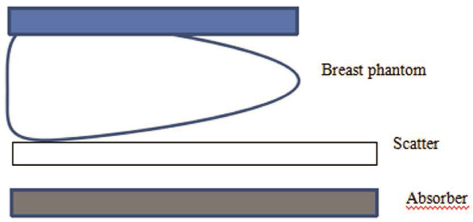

Fig. 1. Single-head Compton camera.

[3]. Ultrasound is an important clinical adjunct technique to mammography by providing details of the lesions. However, these two techniques are not free from limitations. Smith and Andropoulou claim in [4] that despite its effectiveness for detection of breast cancer among women 50 and 69 years old, mammography suffers radiation risk and diminished sensitivity in dense breasts of younger women.

Several new techniques have been developed to improve the preciseness of breast cancer screening. One effective invention is the scintimammography. Scintimammography, known as nuclear medicine breast imaging has the benefit of enabling breast cancer detection among young women by using radioactive materials and electronically collimated camera called Compton camera. The Compton camera utilizes the Compton scattering effect to locate the radioactive source located inside the breast. A Compton camera consists of two parallel-positioned detectors (see Fig. 1). Incident gamma rays are scattered in the scatter detector and subsequently detected by the absorb detector. In both interactions, the energies E_1 and E_2 and positions u_1 and u_2 are recorded. The angle ϕ can be found as follows (e.g. see [5], [6])

$$\cos\phi = 1 - \frac{mc^2 E_1}{(E_1 + E_2)E_2},$$

where m is the mass of the electron and c is the speed of light. The information data u_1, u_2 and ϕ can be used to locate the gamma source somewhere on the cone surface in 3D (see Fig. 2) and the two semilines with common vertex in 2D (see Fig. 3).

Another limitation of the conventional tomography imaging system is its low efficiency when close to the detector edge, which leads to failure in detecting tumors close to the chest wall (e.g. see [7]). New Compton camera, such as C-SMM system, overcomes this problem by placing the detector directly on the chest. It allows the higher efficiency and sensitivity towards the area close to the chest wall [7]. On the other hand, Compton cameras have the flexibility to use a wide range of radio-pharmaceutical energies. In many detecting process, two Compton cameras are used with one below and one above the breast. This system is called the dual-head Compton camera system. In [8], Hruska et al mentioned that the dual-head Compton camera system simultaneously acquires the superior and inferior views, thus it provides the views of the breast in both the craniocaudal position and mediolateral oblique position. Theoretically speaking, dual-head Compton camera is more sensitive in detecting breast tumors than the single-head Compton camera, as it gives two separate images [9]. However, the preliminary results from [8] explained that due to the symmetry position, the results from the two cameras are very similar. Several works, e.g. [7], [8], [9], [10], concentrated on the single-head Compton camera to detect breast cancer (see Fig. 1). In all these works the attenuation phenomena was neglected. However, in scintimammography ignoring the effect of the attenuation of photon can significantly degrade the quality of the reconstruction image. This problem has been recently studied by [11]. The authors studied the attenuated V-line transform in 2 dimensions using circular harmonic expansions and derived an analytic inversion approach in the case of vertices on a circle.

In this paper we introduce the attenuated V-line transform and present some of its new properties on a class of V-lines with vertical central axis and vertex on the x-axis. We start by detailing the problem in section 2 before providing the definitions and recalling the important results about the V-line Radon transform in section 3. At last, section 4 introduces the concept of the attenuated

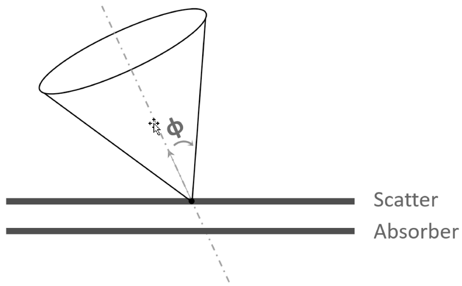

Fig. 2. The Compton camera.

V-line transform in $\mathbb{R}^2$ and present an integral relations between the attenuated V-line transform and the exponential Radon transform.

II. Formulation of the problem

In 2D scintimammography, f represents the distribution of radiotracer concentration inside the breast. Before the γ rays emitted from the radioactive source arrive at the detector, they are attenuated by an attenuation coefficient μ which is a real function on $\mathbb{R}^2$. In the case of uniformly attenuating medium μ can be approximated as a constant in the domain of the function f. Therefore the data may be modeled by a set of exponential weighted V-line integrals of f over two semilines L^+ and L^- with common vertex $(u, 0)$, vertical central axis and a half opening angle ϕ, called attenuated V-line projections $V[f]_\mu(u,\phi)$ (see Fig. 3). After making all the measurements for all possible ϕ and all vertexes $(u, 0)$, one obtains a two-dimensional family of $V[f]_\mu(u,\phi)$ data. The problem of image reconstruction in 2D scintimammography requires the inversion of $V[f]_\mu$, i.e. finding f from the measured data $V[f]_\mu$. Our goal is to present some new properties of the attenuated V-line transform $V[f]_\mu$ on a class of V-lines with vertical central axis and vertex on the x-axis.

III. Notations and preliminaries

Let f, be compactly supported function in the half space $\mathbb{R}\times(0,\infty)\in\mathcal{S}(\mathbb{R}^2)$ and let $\theta = (\cos\phi,\sin\phi)^\intercal \in S^1 := \{\mathbf{v}\in\mathbb{R}^2, |\mathbf{v}| = 1\}$. $V(u,\phi)$ simply consists of two half-lines L^+ and L^- with common vertex $(u, 0)$. We denote by ϕ the half opening angle of the V-lines and $\mu\in\mathbb{R}$ the attenuation constant.

Definition 3.1: The attenuated V-line transform of f at point $(u,\phi)\in\mathbb{R}\times(0,\frac{\pi}{2})$ is defined by

$$V[f]_\mu(u,\phi) = \int_{V(u,\phi)} f(\mathbf{v})e^{\mu l(\mathbf{u},\mathbf{v})}\mathrm{d}l,$$

where $l(\mathbf{u},\mathbf{v}) = |(u,0)-(x,y)|$ is the distance from the vertex $(u, 0)$ to the point with coordinate (x, y), dl is the length element on the V-line $V(u,\phi)$.

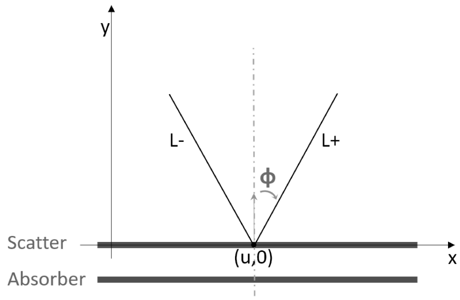

Fig. 3. Geometrical setup of the V-line $V(u,\phi)$.

The V-line transform $V[f](u,\phi)\in\mathbb{R}\times(0,\frac{\pi}{2})$ is thus given by $V[f]_{\mu=0}$ (e.g. see [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22]).

Definition 3.2: The exponential Radon transform $T_\mu f$ on the unit cylinder $Z = S^1\times\mathbb{R}$ is defined by

$$T_\mu f(\theta,s) = \int_{\mathbb{R}} e^{\mu t} f(s\theta + t\theta^\perp)\mathrm{d}t$$

where $\theta^\perp = (-\sin\phi,\cos\phi)\in S^1$.

The classical Radon transform $Rf(\theta,s)\in Z = S^1\times\mathbb{R}$ is thus given by $T_{\mu=0}$.

The Fourier transform generated by a function $f(x, y)$ with respect to the first argument is denoted by

$$\hat{f}(\lambda,y) = \int_{\mathbb{R}} f(x,y)\,e^{-\mathrm{i}\lambda x}\mathrm{d}x$$

where $(\lambda,y)\in\mathbb{R}\times\mathbb{R}$.

IV. The V-line transform and its inversions

The inversion problem of the V-line transform has been studied by many authors. In the following we only recall some of the important results useful for breast cancer detection with Compton cameras.

Theorem 4.1: (Projection-slice theorem for the V-line Radon transform) Consider a function $f \in C^\infty(\mathbb{R}^2)$ compactly supported in the half space $\mathbb{R} \times (0, \infty)$. Then we have

$$\cos\phi\, \widehat{V[f]}(\lambda, \phi) = 2\mathfrak{F}^{(c)}[\widehat{f}](\lambda, \lambda \tan\phi),$$

where $\mathfrak{F}^{(c)}$ is the Fourier-cosine transform with respect to the second argument, $\widehat{f}$ is the Fourier transform of f with respect to the first argument and $\widehat{V[f]}(\lambda, \phi)$ is the Fourier transform of $V[f]$ with respect to the first argument.

Theorem 4.2: Consider a function $f \in C^\infty(\mathbb{R}^2)$ compactly supported in the half space $\mathbb{R} \times (0, \infty)$ $\in \mathcal{S}(\mathbb{R}^2)$. We can write

$$\hat{f}(\lambda, y) = \frac{|\lambda|}{\pi} \int_0^\infty \frac{\cos ty}{\sqrt{t^2+1}}\, \widehat{V[f]}(\lambda, t)\, \mathrm{d}t,$$

where $\hat{f}$ and $\widehat{V[f]}(\lambda, \phi)$ are the Fourier transforms of f and $V[f]$ respectively in the first argument and $t = \tan\phi$.

Proof: See [22], [23]. ∎

An alternative inversion formula is based on the results of the classical Radon transform.

Theorem 4.3: Consider a function $f \in C^\infty(\mathbb{R}^2)$ compactly supported in the half space $\mathbb{R} \times (0, \infty)$ $\in \mathcal{S}(\mathbb{R}^2)$. Then we have

$$V[f](u, \phi) = Rf_s(\theta, u\cos\phi)$$

where f_s is an even extension of f obtained by symmetry with respect to the x-axis and $\theta = (\cos\phi, \sin\phi)^\intercal \in S^1$.

Proof: We can write the V-line transform

$$\begin{aligned} V[f](u, \phi) &= \int_0^\infty f(u + t\sin\phi, t\cos\phi)\mathrm{d}t \\ &\quad + \int_0^\infty f(u - t\sin\phi, t\cos\phi)\mathrm{d}t. \end{aligned}$$

We use the substitution

$$y = t\cos\phi$$

to obtain

$$\begin{aligned} V[f](u, \phi) &= \int_0^\infty f(u + y\tan\phi, y)\frac{\mathrm{d}y}{\cos\phi} \\ &\quad + \int_0^\infty f(u - y\tan\phi, y)\frac{\mathrm{d}y}{\cos\phi}. \end{aligned}$$

We change $y' = -y$ in the first integral

$$\begin{aligned} V[f](u, \phi) &= \int_{-\infty}^0 f(u - y\tan\phi, -y)\frac{\mathrm{d}y}{\cos\phi} \\ &\quad + \int_0^\infty f(u - y\tan\phi, y)\frac{\mathrm{d}y}{\cos\phi}. \end{aligned}$$

Using an approach similar to [24], we consider an even extension obtained by symmetry with respect to the x-axis denoted by f_s:

$$f_s(x, y) = \begin{cases} f(x, y) & 0 \le y \\ f(x, -y) & 0 > y \end{cases}$$

or we can write it as

$$f_s(x, y) = f(x, |y|).$$

Now we can combine the two integrals

$$V[f](u, \phi) = \frac{1}{\cos\phi} \int_{-\infty}^\infty f_s(u - y\tan\phi, y)\mathrm{d}y.$$

We recognize the integral as the integral along the line perpendicular to $(\cos\phi, \sin\phi)^\intercal$ with signed distance $u\cos\phi$ from the origin. In fact, we can explain the above conclusion using the definition of the classical Radon transform,

$$Rf_s(\theta, s) = \int_{\mathbb{R}} f_s(s\theta + t\theta^\perp)\mathrm{d}t,$$

where $(\theta, s) \in S^1 \times \mathbb{R}$, we can write

$$Rf_s\left((\cos\phi, \sin\phi)^\intercal, u\cos\phi\right) = \int_{\mathbb{R}} f_s(u + \cos^2\phi - t\sin\phi, u\cos\phi\sin\phi + t\cos\phi)\mathrm{d}t.$$

With the change of variables

$$y = u\cos\phi\sin\phi + t\cos\phi,$$

we can write

$$
\begin{aligned}
Rf_s&\left((\cos\phi,\sin\phi)^\intercal, u\cos\phi\right)\\
&=\int_{\mathbb{R}} f_s(u-y\tan\phi,y)\frac{\mathrm{d}y}{\cos\phi}\\
&=\frac{1}{\cos\phi}\int_{\mathbb{R}} f_s(u-y\tan\phi,y)\mathrm{d}y.
\end{aligned}
$$

■

Corollary 4.4: An exact solution of the inversion problem for the V-line transform is given by the formula

$$
f_s(\mathbf{v})=\frac{1}{2\pi i}\int_0^\pi \mathcal{H}(\partial_t V[f])\left(\frac{<\mathbf{v}.\theta>}{\cos\phi},\phi\right)\mathrm{d}\phi, \quad (1)
$$

where $\mathcal{H}$ is the Hilbert transform defined by

$$
\mathcal{H}g(t)=\frac{1}{2\pi}\int_{\mathbb{R}} sgn(r)\hat{g}(r)e^{irt}\mathrm{d}r,
$$

$\hat{g}(r)$ is the Fourier transform of $g(t)$ and $sgn(r)$ is the sign function.

Proof: The filtered backprojection formula is used to invert the classical Radon transform (see [25]).

■

We can conclude that the knowledge of the V-line transform can be transferred into the knowledge of the classical Radon transform of f_s the original function and its mirror with respect to x-axis. Using the uniqueness inversion of the classical Radon transform, f_s can be uniquely recovered and consequently f (see [24] for numerical simulations).

V. The attenuated V-line transform

Theorem 5.1: Consider a function $f\in C^\infty(\mathbb{R}^2)$ compactly supported in the half space $\mathbb{R}\times(0,\infty)$. Then we have

$$
\cos\phi\,\widehat{V[f]_\mu}(\lambda,\phi)=\int_{-\infty}^{\infty}\hat{f}(\lambda,|y|)e^{\frac{\mu|y|}{\cos\phi}}e^{-i\lambda y\tan\phi}\mathrm{d}y
$$

where $\widehat{f}$ is the Fourier transform of f with respect to the first argument and $\widehat{V[f]_\mu}(\lambda,\phi)$ the Fourier transform of $V[f]_\mu$ with respect to the first argument.

Proof: $V(u,\phi)$ simply consists of two half-lines L^+ and L^- with common vertex $(u,0)$, so we can write the attenuated V-line transform

$$
\begin{aligned}
V[f]_\mu(u,\phi) &= \int_{L^+} f(x,y)e^{\mu l((u,0),(x,y))}\mathrm{d}l\\
&\quad+\int_{L^-} f(x,y)e^{\mu l((u,0),(x,y))}\mathrm{d}l,\\
V[f]_\mu(u,\phi) &= \int_0^\infty f(u+t\sin\phi,t\cos\phi)e^{\mu t}\mathrm{d}t\\
&\quad+\int_0^\infty f(u-t\sin\phi,t\cos\phi)e^{\mu t}\mathrm{d}t.
\end{aligned}
$$

We use the substitution

$$
y=t\cos\phi
$$

to obtain

$$
\begin{aligned}
V[f]_\mu(u,\phi) &= \int_0^\infty f(u+y\tan\phi,y)e^{\frac{\mu y}{\cos\phi}}\frac{\mathrm{d}y}{\cos\phi}\\
&\quad+\int_0^\infty f(u-y\tan\phi,y)e^{\frac{\mu y}{\cos\phi}}\frac{\mathrm{d}y}{\cos\phi}.
\end{aligned}
$$

We change $y'=-y$ in the first integral

$$
\cos\phi\,V[f]_\mu(u,\phi)=\int_{-\infty}^{\infty} f(u-y\tan\phi,|y|)e^{\frac{\mu|y|}{\cos\phi}}\mathrm{d}y.
$$

This result is further simplified using the Fourier transform with respect to the first argument to obtain

$$
\cos\phi\widehat{V[f]_\mu}(\lambda,\phi)=\int_{-\infty}^{\infty}\hat{f}(\lambda,|y|)e^{\frac{\mu|y|}{\cos\phi}}e^{-i\lambda y\tan\phi}\mathrm{d}y.
$$

■

Theorem 5.2: Consider a function $f\in C^\infty(\mathbb{R}^2)$ compactly supported in the half space $\mathbb{R}\times(0,\infty)$. Then we have

$$
\int_0^\pi V[f]_\mu(u,\phi)\mathrm{d}\phi=\frac{e^{\mu|\mathbf{u}|}}{|\mathbf{u}|}*f(\mathbf{u}),
$$

where $\mathbf{u}=(u,0)$.

Proof:

$$
\begin{aligned}
&\int_0^\pi V[f]_\mu(u,\phi)d\phi\\
&\quad=\int_0^\pi\int_0^\infty f(u+t\sin\phi,t\cos\phi)e^{\mu t}\mathrm{d}t\mathrm{d}\phi\\
&\quad+\int_0^\pi\int_0^\infty f(u-t\sin\phi,t\cos\phi)e^{\mu t}\mathrm{d}t\mathrm{d}\phi.
\end{aligned}
$$

Changing ϕ to $-\phi$ in the second integral and using 2π-periodicity of sine and cosine functions, we obtain

$$\int_0^{\pi} V[f]_\mu(u,\phi)d\phi = \int_0^{2\pi}\int_0^{\infty} f(u+t\cos\phi, t\sin\phi)e^{\mu t}\mathrm{d}t\mathrm{d}\phi.$$

Let $\mathbf{y} = (t\cos\phi, t\sin\phi)$, we get

$$\int_0^{\pi} V[f]_\mu(u,\phi)\mathrm{d}\phi = \int_{\mathbb{R}^2} f(\mathbf{u}+\mathbf{y})e^{\mu|\mathbf{y}|}\frac{\mathrm{d}\mathbf{y}}{|\mathbf{y}|}.$$

We can write it in the convolution form

$$\int_0^{\pi} V[f]_\mu(u,\phi)\mathrm{d}\phi = \frac{e^{\mu|\mathbf{u}|}}{|\mathbf{u}|} * f(\mathbf{u}).$$

This formula is the starting point for reconstruction method of ρ-filtered layergram type, see ([25], Chapter V.6). The authors plan to address this problem in future work.

■

Using the dual operator $T_\mu^\sharp$ as defined

$$T_\mu^\sharp g(\mathbf{u}) = \int_{S^1} e^{\mu\mathbf{u}\cdot\mathbf{x}^\perp} g(\mathbf{x}, \mathbf{u}\cdot\mathbf{x})\mathrm{d}\mathbf{x},$$

we derive a new integral relation between the attenuated V-line transform and the exponential Radon transform.

Corollary 5.3: Consider a function $f \in C^\infty(\mathbb{R}^2)$ compactly supported in the half space $\mathbb{R}\times(0,\infty)$. Then we have

$$\int_0^{\pi} \left(V[f]_\mu + V[f]_{-\mu}\right)(u,\phi)\mathrm{d}\phi = T_{-\mu}^\sharp T_\mu f(\mathbf{u}).$$

Proof:

$$\int_0^{\pi} \left(V[f]_\mu + V[f]_{-\mu}\right)(u,\phi)\mathrm{d}\phi = \left(\frac{2\cosh(\mu|\mathbf{u}|)}{|\mathbf{u}|}\right) * f(\mathbf{u}).$$

$$\int_0^{\pi} \left(V[f]_\mu + V[f]_{-\mu}\right)(u,\phi)\mathrm{d}\phi = T_{-\mu}^\sharp T_\mu f(\mathbf{u}).$$

The last equality is due to ([25], Chapter II.6).

■

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