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Effect of increasing stenosis over time on hemodynamics

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Abstract: **A hard layer, that develops in the inner wall of an artery, makes the blood flow difficult and it can harm the cardiovascular system because of the abnormality in blood supply. The problem becomes worse when the layer gets thicker due to increased deposition over time. The effect of increasing stenosis on flow characteristics in an artery is studied by taking blood as a non-Newtonian fluid. To address the effect of increasing stenosis over time, a non-dimensional temporal term is included in the geometry of stenosis and is applied to derive the flow parameters like velocity profile, volumetric flow rate and pressure drop. The maximum and minimum shear stress ratio and pressure drop ratio are also calculated using the term. The results obtained are analyzed to show the effect of increasing stenosis over time on these flow parameters. Volumetric flow rate, pressure drop and its ratio, and shear stress ratio are compared with the ratio of the thickness of the stenosis and normal artery radius while analyzing the results. It is found that the volumetric flow rate decreases with time, pressure drop and its ratio increases with time, and the shear stress ratio decreases as the time elapses. The result shows that it is appropriate to include the temporal term to understand the effect of increasing stenosis over time on blood flow parameters. The aim of this article is to correct the drawback that evolves while supposing the symmetric shape of the stenosis.**

Keywords: **Radius reducing factor, Increasing stenosis over time, Impedance, Distortion, Aortic stenosis**

I. Introduction

The study of the blood flow through a stenosed artery is of great importance for understanding defects in cardiovascular system. Blood is composed of plasma and suspended blood cells in it. Plasma behaves as a Newtonian fluid, while suspended blood cells are non-Newtonian deformable fluid particles [1, 2]. These suspended particles include red blood cells (RBC), white blood cells (WBC), platelets, proteins, other organic molecules, and salts. RBC occupies biggest volume of about 45% of the blood [3].

Antonova et al. [4] conducted a study on the deformability nature, elasticity, and adhesion force of the erythrocytes while passing through the capillaries and concluded that the erythrocyte elasticity increases significantly in diabetic patients compared to healthy individuals [3]. Blood viscosity lies between 3.5 cP to 5.5 cP, but this value changes according to other hemodynamic conditions [5].

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Normal RBCs align in the direction of flow and follow the tank tread-like motion when shear stress increases. They deform into an elliptical shape following motion [6]. It is generally recognized that irregular intravascular growth causes stenosis to form at different points in the arterial system, reducing the lumen and obstructing blood flow [7]. High risk of death is experienced in a patient with cancer and aortic stenosis [7]. It has been suggested that the mechanical factors have significant role to determine where atherosclerosis occurs, low mean and oscillatory fluid wall shear stresses are also estimated to be responsible for the stenosis [8, 9].

The curvature of artery which varies with time has a significant impact on the flow characteristics. The velocity profile is distorted towards the outer wall of the curved artery, and the maximum distortion occurs roughly halfway between minimum and maximum curvatures [9, 10].

Blood flow in small capillaries depends highly upon deformability of the hematocrits which rises viscosity and a favorable situation for deposition [5, 11–13]. Due to stenosis, radius of the artery decreases gradually which leads the serious circulatory disorder [14]. Depending upon the non-Newtonian nature of the blood, Sankar and Lee [14] have taken blood as a Casson fluid, Punalagusamy and Manchi [15] have taken blood as a K-L Newtonian fluid and Sankar [2] has supposed a Herschel-Bulkley fluid. According to Muravyav et al. [16], sodium nitroprusside and hydrogen sulfide aid in increasing the deformability of erythrocytes in the case of gasotransmitter donors. This, in turn, helps reduce the aggregation of RBCs.

Young [17] has discussed the effect of stenosis and concluded that the effect is significant when it crosses the critical value. The bifurcation region of the carotid artery where there is a low velocity and low wall shear stress is more favorable for plaque buildup that causes the blood arteries to gradually constrict, and may bring ischemic stroke due to this plaque [18].

In four separate examples, including cases with multiple stenosis and without stenosis, Antonova et al. [19] conducted a hemodynamic study in the carotid artery bifurcation and determined velocity profile and wall shear stress.

Young et al. [20] have made a comparative study of the flow parameters in the arteries with and without stenosis in dogs. With the slip velocity at wall, the typical time-dependent stenosis and its impact have been examined by Khan et al. [21]. To maintain normal blood supply, blood pressure rises with increasing stenosis. Shear stress increases due to increase in pressure, and the pressure gradient is greatest at the maximum thickness of the stenosis [22]. Due to the cardiac action, coronary arteries display extremely complicated wall motion patterns, including bending, stretching, twisting, vessel torsion, and vessel displacement, which impact significantly in the transportation of blood cells and oxygen [23].

Lee and Fung [24] have studied the flow in a constricted tube taking steady flow with low Reynolds number. In the stenosed part of an artery, flow variables such as velocity, volumetric flow rate and pressure drop are discussed and respective formulae are derived by Kafle et al. [25]. Antonova et al. [26] have discussed the effects of velocity and vorticity characterstics at the apex of the bifurcation in stenotic region.

Padmanabhan and Jayaraman [13] have studied shear stress and flow resistance due to stenosis in a curved tube with constriction and, concluded that the arterial wall is elastic in nature and less affected by pressure gradient.

Das et al. [27] and Phaijoo [28] have respectively studied the blood flow characteristics in small arteries with curvature and stenosis when catheters of different sizes were inserted and concluded that as catheter size increases, so do the magnitude of pressure drop, impedance, and shear stress.

Santamirana et al. [10] have analyzed the flow pattern in a curved tube whose curvature changes with time, using specific values for the Reynolds number, mean radius of curvature, and deformation parameter.

Schilt et al. [9] have studied the impact of time varying curvature on velocity profile and concluded that the curvature causes the velocity profile to be biased towards outer wall and to fluctuate in form.

Iori et al. [29] have studied the effect of curvature on transport system within arterio-venous fistulae and it is shown that flow pattern of blood and oxygen transport are hugely affected by curvature changing with time.

Yao et al. [30] have studied the combined effect of different angles of curvature, different degrees of stenosis and varying Reynolds numbers. Moreover the secondary flow is significant in the curved portion, which moves towards central portion of the outer wall starting from the inner.

Liu [31] has studied the curved artery with stenosis and its influence on the pulstile flow. The concept of increasing stenosis over time was used by Young [17] to calculate pressure drop and its ratio.

Bravo-Jaimes et al. [7] have studied the patients having both cancer and aortic stenosis and concluded that such patients have high rate of progression in the increment of stenosis.

Above mentioned literature review shows that the blood flow in the stenosed part of an artery has high degree impact in hemodynamics and in cardiovascular disease. Naturally, the stenosis may increase over time and ultimately reach the condition of complete occlusion [7].

Uniformly increasing stenosis reduces the lumen and the ratio of the radii in stenosed and stenosis free region which affects the blood flow parameters. In this work, we incorporate a dimensionless temporal term $e^{-\frac{t}{T}}$ in the geometry of stenosis to measure the decreasing ratio of the radii. This term is used previously by Young [17] to calculate pressure drop and its ratio using blood as a Newtonian fluid.

We find this value quite useful because it can represent the uniform rate of decrement of a quantity. This dimensionless quantity between 0 and 1 can be varied by taking different values of t and a fixed value of T. After changing the geometry, we calculate the flow variables and analyze them.

II. Blood flow model for increasing stenosis over time

In this study, we investigate the effect of continuously increasing stenosis over time in blood flow parameters. To address this issue effectively, we have incorporated a dimensionaless temporal term in the geometry of the stenosis. Solutions for velocity profile, volumetric flow rate, shear stress and pressure drop are obtained after adding the temporal term and the results are analyzed. In this mathematical model, it is assumed that the flow is laminar and axisymmetric in the stenosed region of an artery.

We also assume that the flow is completely developed. Radius of the stenosed artery is R, radius in non-stenosed region is R_0. Velocity is a function of r, and r varies from 0 to R in the stenosed region. Pressure is denoted by p and the pressure gradient is denoted by $P = -\dfrac{\partial p}{\partial z}$. Thickness of increasing stenosis over time is addressed by using a non-dimensional temporal term $e^{-\frac{t}{T}}$. In this term, we have used a fixed time T to make the temporal factor dimensionless, which can be chosen in an appropriate way. The time t in years is to measure the rate of annual increment.

In the geometry of stenosis, we have taken cosine function which is symmetric in nature so the ratio decreases naturally for the interval from $-z_0$ to 0, but after 0, the value of the cosine function decreases and the ratio R/R_0 increases which is opposite of the real situation. To solve this problem, we are using this temporal term in the geometry of the stenosis. When we use appropriate value of the time t, we can address the problem effectively in the interval from 0 to z_0, because this factor will help to measure the decreasing ratio of R/R_0.

A. Geometry of stenosis and velocity for non-Newtonian fluid

We consider blood as a non-Newtonian fluid flowing in a circular tube. Unit control volume is considered inside the tube. Then, the shear stress force in the outer surface of the control volume is

$$\tau = -\mu\left(\frac{dv}{dr}\right)^{1/n},$$

where μ is viscosity, r is variable radius and v is the velocity of the blood. In terms of pressure, shear stress formula can be written as

$$\tau = \frac{Pr}{2}.$$

The pressure gradient $-\dfrac{\partial p}{\partial z}$ is denoted by P which is decreasing gradually from beginning to the end of the stenosis. From these two relations,

$$\frac{dv}{dr} = \left(\frac{Pr}{2\mu}\right)^{1/n}, \tag{1}$$

and on integrating from r to R gives,

$$v = \left(\frac{P}{2\mu}\right)^{1/n} \frac{n}{(n+1)}\left(R^{1+\frac{1}{n}} - r^{1+\frac{1}{n}}\right). \tag{2}$$

At the center of the vessel, $r = 0$ and the velocity is maximum, but decreases gradually towards the wall as r increases. Since we have considered blood as non-Newtonian and power law index n for blood is $0.68 \leq n \leq 0.80$, we have taken $n = 0.75$ as an average value, and geometry of the stenosis is defined as [32]

$$\frac{R}{R_0} = \begin{cases} 1 - \frac{\delta}{2R_0}\left(1 + \cos\frac{\pi z}{z_0}\right), & \text{for } -z_0 \leq z \leq z_0, \\ 1, & \text{for } |z| \geq z_0. \end{cases} \tag{3}$$

As R and R_0 are the radii in the stenosed and non-stenosed region, their ratio becomes one when R and R_0 are equal in stenosis free region. Length and thickness of the stenosis are $2z_0$ and δ, respectively, and z varies from $-z_0$ to z_0.

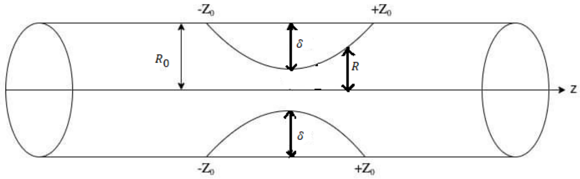


Fig. 1: Part of the artery with stenosis.

When we incorporate the temporal factor in equation (3), it results in

$$\frac{R}{R_0} = \left(1 - \frac{\delta}{2R_0}\left(1 + \cos\frac{\pi z}{z_0}\right)\right)e^{-\frac{t}{T}}. \quad (4)$$

The multiplication factor $e^{-\frac{t}{T}}$ is a nonzero real number between 0 and 1 which represents the uniform decrement of the ratio $\frac{R}{R_0}$. Here t is time measured in year and T is time related constant. At a certain point of stenosis, both z and δ are fixed and the temporal term $e^{-\frac{t}{T}}$ decreases gradually for increasing t. Then the product of the temporal term $e^{-\frac{t}{T}}$ and the expression $\left(1 - \frac{\delta}{2R_0}\left(1 + \cos\frac{\pi z}{z_0}\right)\right)$ decreases gradually, as a result, the ratio $\frac{R}{R_0}$ also decreases which is the effect of the increasing stenosis over time. This quantity $\frac{R}{R_0} \to 0$ for very large t and reaches the condition of complete occlusion. Now we have from equation (2),

$$v = \left(\frac{P}{2\mu}\right)^{1/n}\frac{n}{(n+1)}R_0^{\frac{n+1}{n}}\left(\left(\frac{R}{R_0}\right)^{\frac{n+1}{n}} - \left(\frac{r}{R_0}\right)^{\frac{n+1}{n}}\right) \quad (5)$$

Now substituting the expression for $\frac{R}{R_0}$ from equation (4) in equation (5), we have

$$v = \left(\frac{P}{2\mu}\right)^{1/n}\frac{n}{(n+1)}R_0^{\frac{n+1}{n}}\left(-\left(\frac{r}{R_0}\right)^{\frac{n+1}{n}} + \left(\left(1 - \frac{\delta}{2R_0}\left(1 + \cos\frac{\pi z}{z_0}\right)\right)e^{-\frac{t}{T}}\right)^{\frac{n+1}{n}}\right). \quad (6)$$

After applying binomial expansion formula when the power is not an integer (we have $n = 0.75$) up to two terms, and taking $\cos\frac{\pi z}{z_0}$ equal to 1 as its maximum value, gives

$$v = \left(\frac{P}{2\mu}\right)^{1/n}\frac{n}{(n+1)}R_0^{\frac{n+1}{n}}\left(-\left(\frac{r}{R_0}\right)^{\frac{n+1}{n}} + \left(1 - \frac{(n+1)\delta}{nR_0} + \frac{(n+1)\delta^2}{2n^2R_0^2}\right)e^{-\frac{t(n+1)}{Tn}}\right). \quad (7)$$

Although we do not have data for the rate of increase in thickness of the stenosis, the thickness increases gradually and the radius R decreases. To address this decreasing rate, this non-dimensional temporal term $e^{-t/T}$ is justified. According to Cleaveland clinic [33], the valve area decreases by 0.1 to 0.3 square centimeters per year, but we do not have any data for the rate of increment of the thickness. The increasing rate varies for different age groups and the rate is high for those having age more than 65 years [33]. We have managed the value of $e^{\frac{t}{T}}$ such that the value of $\frac{R}{R_0}$ is reduced yearly by 10% approximately. This reduced space is occupied by the stenosis and the ratio $\frac{\delta}{R_0}$ increases which affects the hemodynamic parameters.

B. Volumetric flow rate with time effect

To calculate the volumetric flow rate Q, we use

$$Q = 2\pi\int_0^R vrdr.$$

After substituting the expression for v from (2),

$$Q = 2\pi\int_0^R \left(\frac{P}{2\mu}\right)^{1/n}\frac{n}{(n+1)}\left(R^{1+\frac{1}{n}} - r^{1+\frac{1}{n}}\right)rdr, \quad (8)$$

and after integrating, we get

$$Q = \frac{n\pi}{3n+1}\left(\frac{P}{2\mu}\right)^{1/n}R_0^{\frac{3n+1}{n}}\left(\frac{R}{R_0}\right)^{\frac{3n+1}{n}}. \quad (9)$$

After expanding up to the second power of δ using binomial expansion, taking maximum value of 1 for both $\cos\frac{\pi z}{z_o}$ and $\cos\frac{2\pi z}{z_o}$, and further simplification gives

$$Q = \frac{n\pi}{3n+1}\left(\frac{P}{2\mu}\right)^{1/n}R_0^{\frac{3n+1}{n}}\left(1 - \frac{3n+1}{n}\left(\frac{\delta}{R_0}\right) + \frac{(3n+1)(2n+1)}{2n^2}\left(\frac{\delta}{R_0}\right)^2\right)e^{-\frac{(3n+1)t}{nT}}. \quad (10)$$

This relation (10) clearly tells us that the volumetric flow rate decreases as the stenosis increases over time, because of the negative power of e.

C. Effect of increasing stenosis over time on pressure gradient

From equation (9),

$$Q = \frac{n\pi}{3n+1}\left(\frac{P}{2\mu}\right)^{1/n}R^{\frac{3n+1}{n}}. \quad (11)$$

From equation (11),

$$P = \frac{2\mu Q^n}{R_0^{3n+1}}\left(\frac{3n+1}{n\pi}\right)^n\left(\frac{R}{R_0}\right)^{-(3n+1)}. \quad (12)$$

The pressure drop ΔP after the blood passes through stenosis is obtained as

$$\Delta P = \int_{-z_0}^{z_0} \frac{2\mu Q^n}{R_0^{3n+1}} \left(\frac{3n+1}{n\pi}\right)^n \left(\frac{R}{R_0}\right)^{-(3n+1)} dz. \quad (13)$$

Substituting the expansion of $\frac{R}{R_0}$ in the geometry of the stenosis and after binomial expansion,

$$\left(\frac{R}{R_0}\right)^{-(3n+1)} = \left(1 + (3n+1)\left(\frac{\delta}{R_0}\right) + \frac{(3n+1)(3n+2)}{2}\left(\frac{\delta}{R_0}\right)^2\right) e^{\frac{(3n+1)t}{T}}. \quad (14)$$

Substituting equation (14) in equation (13) and after integration, we have

$$\Delta P = \frac{4\mu z_0 Q^n}{R_0^{3n+1}} \left(\frac{3n+1}{n\pi}\right)^n \left(1 + (3n+1)\left(\frac{\delta}{R_0}\right) + \frac{(3n+1)(3n+2)}{2}\left(\frac{\delta}{R_0}\right)^2\right) e^{\frac{(3n+1)t}{T}}. \quad (15)$$

In absence of stenosis, both δ and t are 0. Then the pressure drop $(\Delta P)_0$ becomes

$$(\Delta P)_0 = \frac{4\mu z_0 Q^n}{R_0^{3n+1}} \left(\frac{3n+1}{n\pi}\right)^n. \quad (16)$$

From equation (15) and (16), the pressure drop ratio can be expressed as

$$\frac{\Delta P}{(\Delta P)_0} = \left(1 + (3n+1)\left(\frac{\delta}{R_0}\right) + \frac{(3n+1)(3n+2)}{2}\left(\frac{\delta}{R_0}\right)^2\right) e^{\frac{(3n+1)t}{T}}. \quad (17)$$

D. Shear stress and its ratio of maximum and minimum

The shear stress at the wall of the stenosis is considered as minimum, because it is nearer from the center and is defined as

$$\tau_{\min} = \frac{PR}{2}. \quad (18)$$

Similarly, the shear stress at the wall of the normal artery (without stenosis) is considered maximum because its distance from the center of the artery is highest and is defined as

$$\tau_{\max} = \frac{PR_0}{2}. \quad (19)$$

So, the shear stress ratio is expressed as

$$\frac{\tau_{\max}}{\tau_{\min}} = \frac{R_0}{R} = \left(\frac{R}{R_0}\right)^{-1} = \left(1 + \frac{\delta}{R_0} + \frac{\delta^2}{2R_0^2}\right) e^{\frac{t}{T}}. \quad (20)$$

III. Results and Discussion

Here we analyze different hemodynamic parameters by using various expressions in the previous sections, especially after inclusion of the temporal term.

A. Effect of time and viscosity on velocity

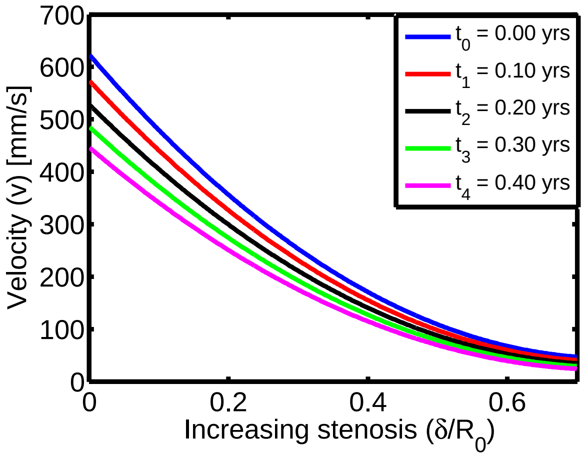
Figure 2a shows the effect of increasing stenosis over time on velocity, keeping the blood viscosity constant. In this case, artery radius is taken 6 mm, blood viscosity 4.5 cP, pressure 100 mm of Hg, value of n is 0.75 and this velocity is measured at a distance of 1.5 mm from the center towards wall of the stenosed artery. Initially when $t = 0.00$, the velocity is 619 mm s^{-1}. For 0.10 year, the velocity is 570 mm s^{-1} similarly for the time 0.20, 0.30 and 0.40 years, the approximate velocities are 524 mm s^{-1}, 480 mm s^{-1} and 443 mm s^{-1} respectively. This shows that the velocity is decreasing gradually as the stenosis increases over time.

Figure 2b shows the effect of increasing blood viscosity on velocity. Parameters are same as in Figure 2a, The distance from the center is taken 1.0 mm only to minimize the effect of shear stress. According to Nader et al. [5], blood viscosity lies in between 3.50 cP to 5.50 cP so the viscosity is taken within this range. For the viscosity 3.50 cP, the velocity is 275 mm s^{-1} and for the viscosities 4.00 cP, 4.50 cP, 5.00 cP and 5.50 cP, the velocities are 231 mm s^{-1}, 198 mm s^{-1}, 172 mm s^{-1} and 151 mm s^{-1} respectively. This shows that the velocities are decreasing gradually with increasing viscosity as the height of the stenosis increases over time to increase the ratio $\frac{\delta}{R_0}$. As the ratio becomes more than 0.70, velocity becomes minimum and the effect of viscosity becomes negligible.

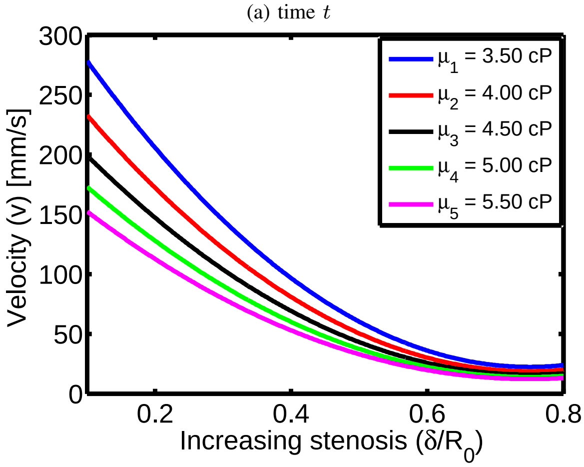
B. Effect of time and viscosity on volumetric flow rate

Figure 3a describes the effect of time on volumetric flow rate for increasing stenosis over time, keeping the blood viscosity constant. Value of t is managed in such a way that the increment of the stenosis is about 10 %. It is not possible to take fixed t for all, because the rate of increment of stenosis is different for different age groups. Radius of the artery in the stenosed part is considered as 6 mm, viscosity μ as 4.5 cP, and pressure as 100 mm of Hg. Volumetric flow rate is measured when the ratio $\frac{\delta}{R0}$ is 0.72 and it can be seen clearly that the ratio decreases gradually as the time increases.

When t is 0.00 years, 0.10 years, 0.20 years, 0.30 years and 0.40 years, the corresponding volumetric flow rate is 64.85 mm^3 s^{-1}, 56.13 mm^3 s^{-1}, 48.58 mm^3 s^{-1}, 42.05 mm^3 s^{-1} and 36.39 mm^3 s^{-1} respectively. The volumetric flow rate is not changing rapidly before

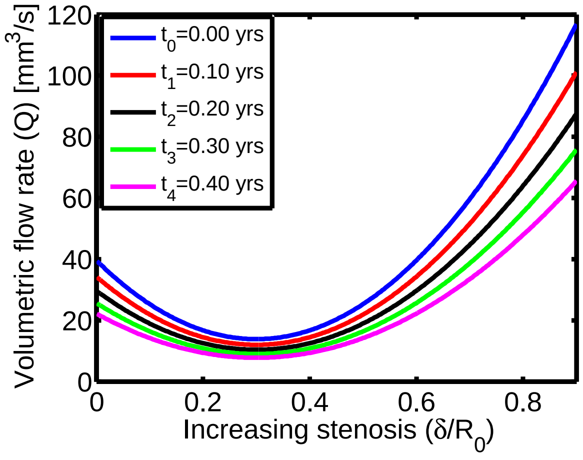


(a) time t

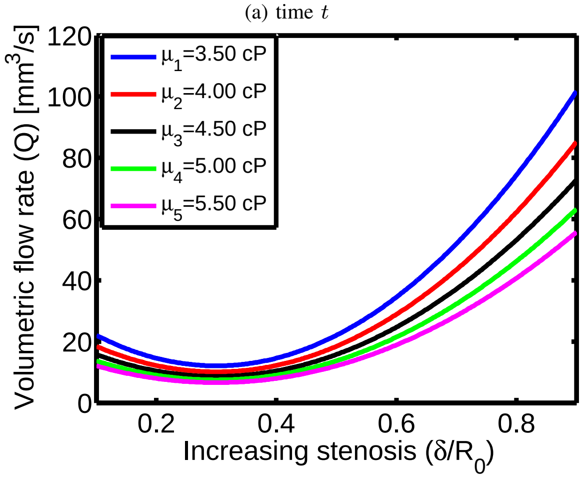


(b) dynamic viscosity μ

Fig. 2: Relation between velocity and increasing stenosis over time for different values of (a) time t and (b) dynamic viscosity μ.



(a) time t



(b) dynamic viscosity μ

Fig. 3: Relation between volumetric flow rate and increasing stenosis over time for different values of (a) time t and (b) dynamic viscosity μ.

the ratio $\frac{\delta}{R_0}$ is 0.30 which indicates that there is no significant change in the volumetric flow rate for mild stenosis. As the ratio $\frac{\delta}{R_0}$ increases from 0.45, the volumetric flow rate is decreasing rapidly as the time increases which shows the significance of time on volumetric flow rate. The volumetric flow rate is decreasing with the constant ratio of 0.8655 approximately for equal amount of time interval, within this range. This uniform ratio may be useful to estimate the nearest value of volumetric flow rate and the ratio $\frac{\delta}{R_0}$ which helps to predict the size of the stenosis. The graph moves downward at the beginning because of the nature of the exponential function and gradually moves upward to indicate accumulated volume. Analysis of the result is expressed in Table I for different time.

Figure 3b describes the effect of viscosity μ on the volumetric flow rate Q as the stenosis is increasing, keeping the time constant. All the parameters are same as in Figure 3a and fixed time 0.20 years is taken in this case. As the viscosity increases from 3.50 cP to 4.00 cP, the volumetric flow rate decreases from 67.26 mm^3 s^{-1} to 56.29 mm^3 s^{-1}. Similarly for the viscosities 4.50 cP, 5.00 cP and 5.50 cP, the corresponding volumetric flow rates are 48.11 mm^3 s^{-1}, 41.80 mm^3 s^{-1} and 36.82 mm^3 s^{-1} respectively. These values are measured when the ratio $\frac{\delta}{R_0}$ is 0.77. The volumetric flow rate decreases almost uniformly as the viscosity increases which concludes that the viscosity of the blood affects the volumetric flow rate significantly and this result is analyzed numerically in Table II.

Table I: Analysis of volumetric flow rate for different time and constant viscosity.

Index i	Time t_i [year]	Volumetric flow rate Q_i	Q_{i+1}/Q_i	$\frac{Q_0-Q_i}{Q_0}$
0	0.00	64.85	—	—
1	0.10	56.13	0.8655	0.1345
2	0.20	48.58	0.8655	0.2509
3	0.30	42.05	0.8655	0.3516
4	0.40	36.39	0.8654	0.4389

Table II: Analysis of volumetric flow rate for different viscosity with constant time.

Index i	Viscosity μ_i	Volumetric flow rate Q_i	Q_{i+1}/Q_i	$\frac{Q_0-Q_i}{Q_0}$
0	3.50	67.26	—	—
1	4.00	56.29	0.8369	0.1631
2	4.50	48.11	0.8546	0.2847
3	5.00	41.80	0.8689	0.3785
4	5.50	36.82	0.8809	0.4526

In Table I, we have analyzed the effect of time on volumetric flow rate. When the time increases uniformly keeping other parameters fixed, the volumetric flow rate Q decreases uniformly. Comparing Q_1 with Q_0 the decrement is about 13.45%. Similarly the decrements are 25.09%, 35.16% and 43.89% when Q_2, Q_3 and Q_4 are compared with Q_0 respectively. In fact, thickness of the stenosis increases with the time and this increment reduces the volumetric flow rate. This table clearly expresses the effect of thickness of the stenosis increasing over time on the volumetric flow rate.

In Table II time is kept constant and viscosity is increased gradually and it is shown that the volumetric flow rate is decreasing uniformly. For the fixed time 0.20 years, the viscosities are increased by 0.50 cP in each step. For this equal increment of viscosity, corresponding volumetric flow rate is shown in the second column. Ratio of the volumetric flow rates after increasing the viscosity by 0.50 cP is listed in the fourth column.

In the last column, volumetric flow rates for different viscosities are compared with the initial viscosity 3.5 cP. Comparing Q_1 with Q_0, it is decreased by 16.31%. Similarly the decrement is 28.47%, 37.85% and 45.26% when Q_2, Q_3 and Q_4 are compared with Q_0 respectively. This shows that the result is reliable and effect of viscosity is also significant.

The comparison of Table I and Table II tells us that the effect of time is slightly more significant than viscosity in the sense of uniformity.

Table III: Analysis of pressure drop for variable time values.

Index i	Time t_i [year]	Pressure drop ΔP_i	$\Delta P_i/\Delta P_{i+1}$	$\frac{\Delta P_i-\Delta P_0}{\Delta P_0}$
0	0.00	47.43	—	—
1	0.10	52.86	0.8973	0.1144
2	0.20	58.91	0.8973	0.2420
3	0.30	65.65	0.8973	0.3841
4	0.40	73.16	0.8973	0.5424

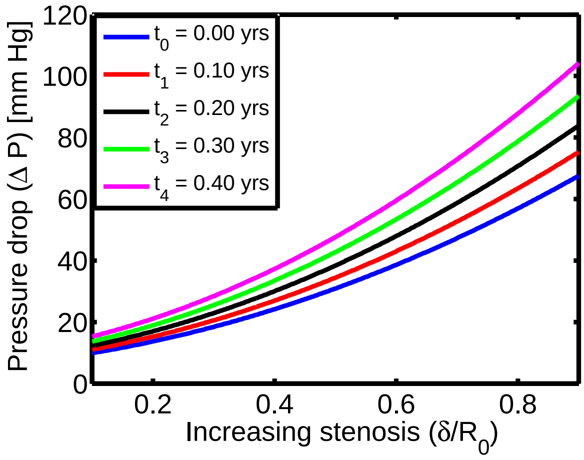
Table IV: Analysis of pressure drop ratio for different time. Last column shows percentage increment from initial value.

Index i	Time t_i [year]	Pressure drop ratio $\frac{\Delta P}{(\Delta P)_0}$	$\frac{\text{Ratio}_{i+1}}{\text{Ratio}_i}$	percentage increment
0	0.00	6.685	—	—
1	0.10	7.450	1.1144	11.44
2	0.20	8.302	1.1144	24.19
3	0.30	9.252	1.1144	38.39
4	0.40	10.310	1.1144	54.22

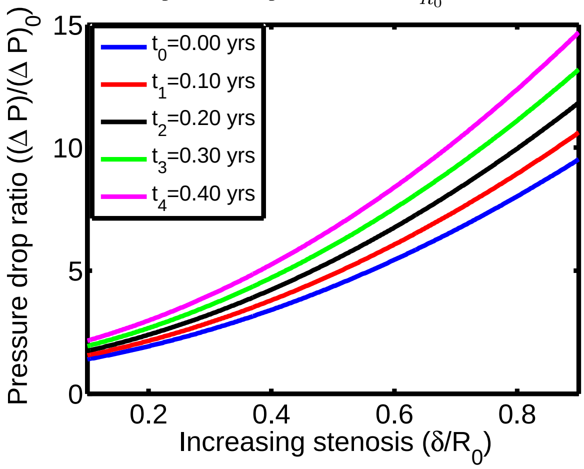
C. Effect of increasing stenosis over time on pressure drop and its ratio

Figure 4a describes the relationship between the pressure drop ΔP and increasing stenosis over time at different levels of stenosis. The radius of the artery in the stenosed part is taken as 6 mm, the viscosity is 4.5 cP, and the pressure is 100 mm of Hg. The pressure drop is measured when the ratio is 0.7. The ratio $\frac{\delta}{R_0}$ is 0 when there is no stenosis and gradually increases to 1 as the stenosis increases to reach the stage of complete occlusion. For the time 0.00 years, 0.10 years, 0.20 years, 0.30 years, and 0.40 years, the corresponding pressure drop values are 47.43, 52.86, 58.91, 65.65, and 73.16 mm of Hg, respectively. The increment in the pressure drop is slow before the ratio $\frac{\delta}{R_0}$ reaches 0.40, but it increases rapidly after this point. The ratio of the pressure drop with the previous value is constant and is approximately 1.1144, which is shown in Table III. Because of this uniformity in the increment, we can use this idea to estimate the neighbouring values, which may further help to measure the size of the stenosis.

Figure 4b describes the relationship between the pressure drop ratio $\frac{\Delta P}{(\Delta P)_0}$ and the ratio $\frac{\delta}{R_0}$ of increasing stenosis for different values of t. Pressure, radius, viscosity, and volumetric flow rate are the same as in Figure 4a. The data for the pressure drop ratio, which are given below are measured when the ratio $\frac{\delta}{R_0}$ is 0.7. Table IV shows that the pressure drop ratio increases uniformly with the ratio $\frac{\delta}{R_0}$. For the time 0.00 years, 0.10 years, 0.20 years, 0.30 years, and 0.40 years, the corresponding pressure drop ratios are



(a) pressure drop ΔP and ratio $\frac{\delta}{R_0}$



(b) pressure drop ratio $\frac{\Delta P}{(\Delta P)_0}$ and ratio $\frac{\delta}{R_0}$

Fig. 4: Relation between (a) pressure drop (b) pressure drop ratio and the ratio of the thickness of the stenosis to the normal artery $\frac{\delta}{R_0}$, for different values of time.

6.685, 7.450, 8.302, 9.252, and 10.310 respectively. The uniform increasing ratio with the previous value is 1.1144 approximately as in Figure 4a. This uniform ratio is quite interesting and may be a subject for further research.

Table III is the quantitative interpretation of Fig. 4a. The pressure drop increases gradually, keeping the ratio constant with the previous pressure drop. In the last column of Table III, we have shown the percentage increment in pressure drop compared with the initial pressure drop $(\Delta P)_0$. When we take four equal time intervals (0.0 to 0.1, 0.1 to 0.2, 0.2 to 0.3, and 0.3 to 0.4 years), the differences in pressure drops are 5.43, 6.04, 6.74, and 7.51. So the pressure drop increases gradually with time for increasing stenosis, even though the time interval is equal. As the time increases from 0.00 years to 0.40 years, the pressure drop increases by 42.80 mm of Hg (from 47.43 to 73.16). The percentage increment compared to the initial value is shown in the last column which indicates that there is a good relationship between increasing stenosis over time and pressure drop.

Table IV describes Figure 4b quantitatively which shows that as time increases by an equal amount, the pressure drop ratio also increases uniformly within the given time range. As the time increases from 0.00 years to 0.40 years, the pressure drop ratio increases from 6.685 to 10.31, and the increment is 3.625, and in percentage, this increment is equal to 54.22%. These data are measured at the point when the ratio $\frac{\delta}{R_0}$ is 0.70. An interesting result is that in every step, the ratio to the previous one is equal, which is 1.1144. The last column shows the gradual increment in the percentage compared with the initial value.

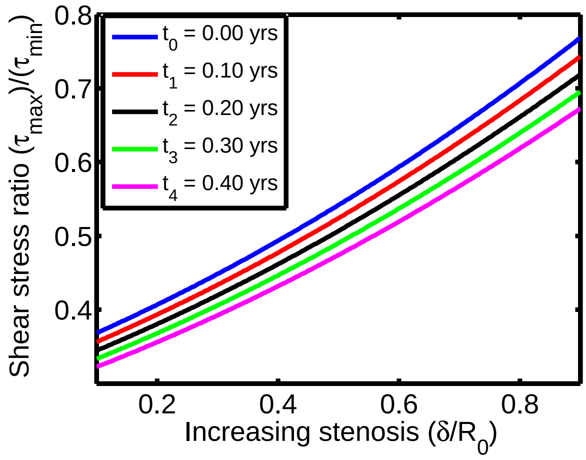


Fig. 5: Relation between the shear stress ratio and increasing ratio of stenosis.

Table V: Analysis of shear stress ratio for different time. Last column shows percentage increment from initial value.

Index i	Time t_i [year]	shear stress ratio $\frac{\tau_{\max}}{\tau_{\min}}$	$\frac{\text{Ratio}_i}{\text{Ratio}_{i+1}}$	percentage increment
0	0.00	0.6488	—	—
1	0.10	0.6275	1.0339	0.0328
2	0.20	0.6069	1.0339	0.0645
3	0.30	0.5870	1.0339	0.0952
4	0.40	0.5678	1.0339	0.1248

D. Effect of increasing stenosis over time on shear stress ratio

Figure 5 describes the relation between shear stress ratio and the ratio of δ/R_0 as the stenosis increases gradually. While taking ratio almost all the parameters cancel except δ, t, T and R_0. The figure shows that the shear stress ratio increases gradually as the ratio δ/R_0 increases. For the time 0.00 years, 0.10 years, 0.20 years, 0.30 years and 0.40 years, the corresponding shear stress ratios are 0.6488, 0.6275, 0.6069, 0.5870 and 0.5678 respectively and the uniform decreasing ratio is 0.9671. These values are measured when the ratio $\frac{\delta}{R_0}$ is 0.7. The major outcome in these relations is that the change (either increment or decrement) is uniform in all the cases.

In Table V, we have analyzed the shear stress ratio for different values of time. The result shows that the shear stress ratio is decreasing slightly but uniformly as the time increases. In each step, it decreases with a constant ratio of 0.9671. In the last column, we have calculated the percentage decrement in shear stress as the time increases. The result shows that the percentage decrement is 3.28%, 6.45%, 9.52% and 12.48% respectively from the initial value as the time interval is increased by 0.10 in each step. This uniformity of shear stress ratio in Table V is also noteworthy.

IV. Summary

We studied the blood flow parameters in a stenosed artery taking blood as a non-Newtonian fluid. Time affected radius reducing factor named as temporal term has been used in the geometry of the stenosis. The analytical solutions of the blood flow parameters are derived after introducing the temporal term to address the reducing ratio $\frac{R}{R_0}$ as the stenosis increases.

The results were simulated and analyzed for the blood flow parameters. The results show that the volumetric flow rate decreases uniformly as the time increases. The pressure drop increases uniformly for equal time intervals. The pressure drop ratio also shows the behavior identical to pressure drop. The shear stress ratio also decreases uniformly with increase in time. The results show various effects of increasing stenosis over time upon the flow parameters for different values of time.

The increasing or decreasing ratio of flow parameters is uniform for equal amount of time intervals, which may be quite useful for the further study. The major application of the uniform change is that the nearby values of the flow parameters can be estimated using these results. It can be useful in the area of medical science, to predict the effect of cardiovascular stenosis in hemodynamics.

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