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Analysis of hemodynamic parameters on two-layered blood flow in a curved artery

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Abstract: **Arterial stenosis is the thickening of the arterial wall due to the growth of aberrant tissues that prevent adequate blood flow in the human circulatory system and induces cardiovascular diseases. Mild stenosis, may lead to serious or permanent damage if remains uncured. There are differences in the curvature response and material composition between the outer layers and the core. There are several locations in blood vessels where they are curved, which affects the blood flow and shear stress. The Navier-Stokes equation in the cylindrical polar coordinate system has been extended by incorporating curvature term in two-layered blood flow along the axial direction with appropriate boundary conditions. Mathematical expressions for hemodynamic parameters such as velocity profile, volumetric flow rate, pressure drop, and shear stress have been calculated analytically in the case of a curve artery with stenosis. Moreover, we have analyzed the effect of stenosis on different hemodynamic parameters with the variation of core and peripheral-layer viscosity and curvature. Flow quantities are affected by the habitancy of stenosis and stipulate different blood flow behavior in both layers in the case of curved artery. This modeling technique may help researchers in medicine, mathematical biology, and bio-engineering.**

Keywords: **arterial stenosis, velocity profile, volumetric flow rate, pressure drop, shear stress, two layered, curved artery**

I. INTRODUCTION

Stenosis is the development of a thick, hard coating in the inner wall of an artery as a result of the infiltration and growth of fatty particles like cholesterol [1–3]. Most often, nutrition and chemistry play a major role in the development of atherosclerosis, and genetics may also have a significant role [4]. Initiating early in life, Stenosis tends to exacerbate with age, impeding circulatory function and causing issues like cerebral strokes, heart failure, and paralysis [4–7]. It frequently manifests in the curved part of the coronary, carotid, abdominal, and pelvic arteries, which often occurs at low shear rates and at high viscosity [8, 9]. Blood pressure increases as stenosis develops to maintain the blood supply [5, 10]. The main factors affecting blood flow in the stenosed portion of the artery include the deformability of erythrocytes, wall shear stress, and viscosity of the blood [6, 11].

In the beginning blood flow was studied by treating it as a single layer, but due to the presence of an inner core layer of erythrocytes and an outer viscous layer of plasma, it is realized that the blood flow can be explained better by using two-layer model [4, 12, 13].

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The material composition of the core and outer layers differ and the curvature responds differently in each layer [14]. Since velocity, viscosity, and shear stress fluctuate differently in the core and peripheral layer, the two-layer approach is better suited to determine the flow characteristics. Blood vessels exhibit curvature in several different places, which changes their velocity and shear stress and affects the blood flow [15].

Local variations in curvatures over time and space impact the velocity profile, contributing to a range of flow patterns, such as constant and phasic flow, within the curved portion of the artery [16, 17]. Stenosis is more likely in curved vessel segments, emphasizing the need for a two-layer blood flow model that incorporates curvature effects to better describe blood flow through stenosis.

Lee and Fung [4] studied the one layer model in a cylindrical coordinate system of the Navier-Stokes equations. Nosovitsky et al. [16] studied the impact of curvature on wall shear stress.

Pokharel et al. [18] used Navier-Stokes equations in a cylindrical polar coordinate system and studied the blood flow parameters by considering the flow is steady, axially symmetrical, fully developed, and laminar.

Dhange et al. [19] investigated, tangent stress pressure is created by a stenotic blood vessel, which reduces the arterial side and generates an aneurysm.

Misra and Chakravarty [20] examined the blood flow in a stenotic portion of an artery by modeling the artery as an initially stressed orthotropic elastic tube filled with a viscous, incompressible fluid. According to Padmanabhan and Jayaraaman [21] due to the combination of stenosis and curvature, there are a number of remarkable fluid mechanical phenomena.

Kafle et al. [6] examined the relationship between curvature and viscosity, pressure drop, velocity distribution, and shear stress ratio when the flow is along z-axis.

Dash et al. [14] derived an appropriate analytic solution to the problem of blood flow through a catheterized artery with a small curvature and mild stenosis, and shown that the effect of curvature is important in blood flow system.

Ponalagusamy and Manchi [7] analyzed the effects of mild stenosis in a two-layered blood flow model, considering various geometries. Musad [1] has calculated the effect of wall shear stress in peripheral and core layer separately in two-layer flow model.

Two-layer pulsatile blood flow model have been studied by Devjyoti and Chakravarty [22] to calculate radius of the stenosed artery, wall shear stress, volume flow rate, viscosity, and pressure drop taking core layer as a Bingham plastic.

Singh et al. [13] have used a two-layer model to determine the volumetric flow rate, pressure, shear stress, and flow resistance.

Chaturani and Kaloni [23] described the two-layer model with couple stress at the core layer taking blood as incompressible fluid.

Srivastava and Rastogi [24] examined the impact of a non-symmetrical shape of stenosis on a two-phase macroscopic flow model.

Bugliarello and Sevilla [12] had shown experimentally that the velocity profile substantially reduces near the inner wall when the diameter is smaller than 50 μm.

Ponalagusamy and Manchi [7] have studied the mild stenosis using two-layer model with geometry for each of the six categories.

Ponalagusami [15] has investigated a mathematical model for blood flow in stenosed arteries with variable slip at the wall and axially variable peripheral layer thickness. The peripheral and core layer viscosities μ_p and μ_c take $(1.4, 1.5, 1.6, 1.7)$ gram mm^{-1} s^{-1} and $(3.0, 3.5, 4.0, 4.5)$ gram mm^{-1} s^{-1} receptively, but this value changes according to other hemodynamic conditions [25]. The model consists of a peripheral layer around a core, with the assumption that the fluids in both regions have different viscosities and are Newtonian in nature.

This article integrates the curvature term into the momentum balance equation along the z-axis to examine blood flow characteristics under different positions and heights of stenosis. The two-layer model, accounting for curvature effects, explores velocity profiles, volumetric flow rates, pressure drops, and shear stresses in various scenarios involving stenosis, curvature, and viscosity.

II. Extended model equation for two-layered blood flow in a curved artery

We are taking into consideration steady flow in an axially symmetrical stenosed artery. The artery is considered as an inelastic circular tube. The radial flow is neglected and blood is assumed as incompressible and Newtonian. Let R and R_0 denote the radii with and without stenosis, respectively. Let r be the radial distance, p pressure, and v^r, v^θ, and v^z are the velocities the radial, perpendicular, and along axial direction. The system can be expressed in cylindrical polar coordinate form (r, θ, z). The equation of continuity and momentum with the curvature effect in z-axis are [26, 27]:

$$\frac{\partial v^r}{\partial r} + \frac{v^r}{r} + \frac{1}{r}\frac{\partial v^\theta}{\partial \theta} + \frac{\partial v^z}{\partial z} = 0, \tag{1}$$

$$\rho\Big(\frac{\partial v^r}{\partial t}+v^r\frac{\partial v^r}{\partial r}+\frac{v^\theta}{r}\frac{\partial v^r}{\partial \theta}-\frac{(v^\theta)^2}{r}+v^z\frac{\partial v^r}{\partial z}\Big) = -\frac{\partial p}{\partial r}+\mu\Big(\frac{1}{r}\frac{\partial}{\partial r}\Big(\frac{\partial}{\partial r}(rv^r)\Big) + \frac{1}{r^2}\frac{\partial^2 v^r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v^\theta}{\partial \theta}+\frac{\partial^2 v^r}{\partial z^2}\Big), \tag{2}$$

$$\rho\Big(\frac{\partial v^\theta}{\partial t}+v^r\frac{\partial v^\theta}{\partial r}+\frac{v^\theta}{r}\frac{\partial v^\theta}{\partial \theta}+\frac{v^r v^\theta}{r}+v^z\frac{\partial v^\theta}{\partial z}\Big) = -\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\Big(\frac{1}{r}\frac{\partial}{\partial r}\Big(\frac{\partial}{\partial r}(rv^\theta)\Big) + \frac{1}{r^2}\frac{\partial^2 v^\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v^r}{\partial \theta}+\frac{\partial^2 v^\theta}{\partial z^2}\Big), \tag{3}$$

$$\rho\Big(\frac{\partial v^z}{\partial t}+v^r\frac{\partial v^z}{\partial r}+\frac{v^\theta}{r}\frac{\partial v^z}{\partial v^\theta}+v^z\frac{\partial v^z}{\partial z}\Big) = -\frac{\partial p}{\partial z}+\mu\Big(\frac{1}{r}\frac{\partial}{\partial r}\Big(r\frac{\partial v^z}{\partial r}+\frac{\kappa P}{16\mu}\Big(R^2-r^2\Big)^2\Big) + \frac{1}{r^2}\frac{\partial^2 v^z}{\partial \theta^2}+\frac{\partial^2 v^z}{\partial z^2}\Big), \tag{4}$$

where κ is the dynamic curvature along the axial direction. We assume that in the axisymmetric flow we have $v^\theta = 0$. For the steady flow of blood, viscosity μ and density ρ are considered to be constant. Then, mass balance and momentum balance equation (1)-(4) are reduced to:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv^r)+\frac{\partial}{\partial z}(v^z)=0 \tag{5}$$

$$\rho\Big(v^r\frac{\partial v^r}{\partial r}+v^z\frac{\partial v^r}{\partial z}\Big) = -\frac{\partial p}{\partial r}+\mu\Big(\frac{\partial^2 v^r}{\partial r^2}+\frac{\partial^2 v^r}{\partial z^2} + \frac{1}{r}\frac{\partial v^r}{\partial r}-\frac{v^r}{r^2}\Big) \tag{6}$$

$$\rho\Big(v^r\frac{\partial v^z}{\partial r}+v^z\frac{\partial v^z}{\partial z}\Big) = -\frac{\partial p}{\partial z}+\mu\Big(\frac{\partial^2 v^z}{\partial r^2}+\frac{\partial^2 v^z}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\Big(v^z+\frac{\kappa P}{16\mu}(R^2-r^2)^2\Big)\Big). \tag{7}$$

We have considered axially symmetrical flow along z-axis only, so $v^r = 0$ and $v^z = v(r)$, which are velocity component parallel to z-axis, then equations (5)-(7) become:

$$\frac{\partial v}{\partial z}=0,\quad 0=-\frac{\partial p}{\partial r},$$
$$0=-\frac{\partial p}{\partial z}+\mu\Big(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{\kappa P}{4\mu}(R^2-r^2)\Big). \tag{8}$$

Suppose pressure drop term as $P = -\partial p/\partial z$, then equation (8) reduces to:

$$-P\frac{r}{\mu}=\frac{\partial}{\partial r}\Big(r\frac{\partial v}{\partial r}+\frac{\kappa P}{16\mu}\Big(R^2-r^2\Big)^2\Big). \tag{9}$$

Here blood is represented by two-layered, inner core and outer peripheral layer with different hemodynamic parameters. Suppose R_s^p is the radius of artery in peripheral layer and R_s^c is the radius of artery in the core layer in presence of stenosis such that $R_s^c = \beta R_s^p$, $R_0^c = \beta R_0^p$, where R_0^p and R_0^c are the radius of peripheral and core layer of normal artery respectively and β is the ratio of core radius to peripheral radius which is called scaling model parameter.

For two-layered model, we divide whole region into peripheral layer as $R_s^c \le r \le R_s^p$ and core region as $0 \le r \le R_s^c$. Assume μ_p and μ_c are viscosity of peripheral and core layer respectively. More precisely two-layered viscosity μ is defined as:

$$\mu=\begin{cases}\mu_p, & \text{at}\ \ R_s^c\le r\le R_s^p,\\ \mu_c, & \text{at}\ \ 0\le r\le R_s^c.\end{cases}$$

Suppose δ_p and δ_c are the maximum height of stenosis in the peripheral and core layer such that $\delta_c = \beta\delta_p$. The geometry of the stenosis in peripheral layer is given by [10]:

$$R_s^p=\begin{cases}R_0^p-\frac{\delta_p}{2}(1+cos\frac{\pi z}{z_0}) & \text{at}\ \ |z|\le z_0,\\ R_0^p & \text{at}\ \ |z|>z_0.\end{cases}$$

Similarly, the geometry of the stenosis in core layer as given by [10]:

$$R_s^c=\begin{cases}R_0^c-\frac{\delta_c}{2}(1+cos\frac{\pi z}{z_1}) & \text{at}\ \ |z|\le z_1,\\ R_0^c & \text{at}\ \ |z|>z_1.\end{cases}$$

The equation (9) for peripheral and core layer, respectively, can be rewritten as:

$$-P\frac{r}{\mu_p}=\frac{\partial}{\partial r}\Big(r\frac{\partial v_p}{\partial r}+\frac{\kappa P}{16\mu_p}\Big((R_s^p)^2-r^2\Big)^2\Big), \tag{10}$$

$$-P\frac{r}{\mu_c}=\frac{\partial}{\partial r}\Big(r\frac{\partial v_c}{\partial r}+\frac{\kappa P}{16\mu_c}\Big((R_s^c)^2-r^2\Big)^2\Big), \tag{11}$$

with boundary conditions [10,27,28]:

$$v_p=\begin{cases}v_c, & \text{at}\ \ r=R_s^c,\\ 0, & \text{at}\ \ r=R_s^p,\end{cases}$$

and $\frac{\partial v_p}{\partial r}=0$, $\frac{\partial v_c}{\partial r}=0$ at $r=0$.

A. Two-layered velocity profile of blood flow in curved artery

1) Peripheral layered velocity: Let v_p be the peripheral velocity (i.e., region $R_s^c \le r \le R_s^p$), the equation (10) becomes:

$$-P\frac{r}{\mu_p}=\frac{\partial}{\partial r}\Big(r\frac{\partial v_p}{\partial r}+\frac{\kappa P}{16\mu_p}\Big((R_s^p)^2-r^2\Big)^2\Big).$$

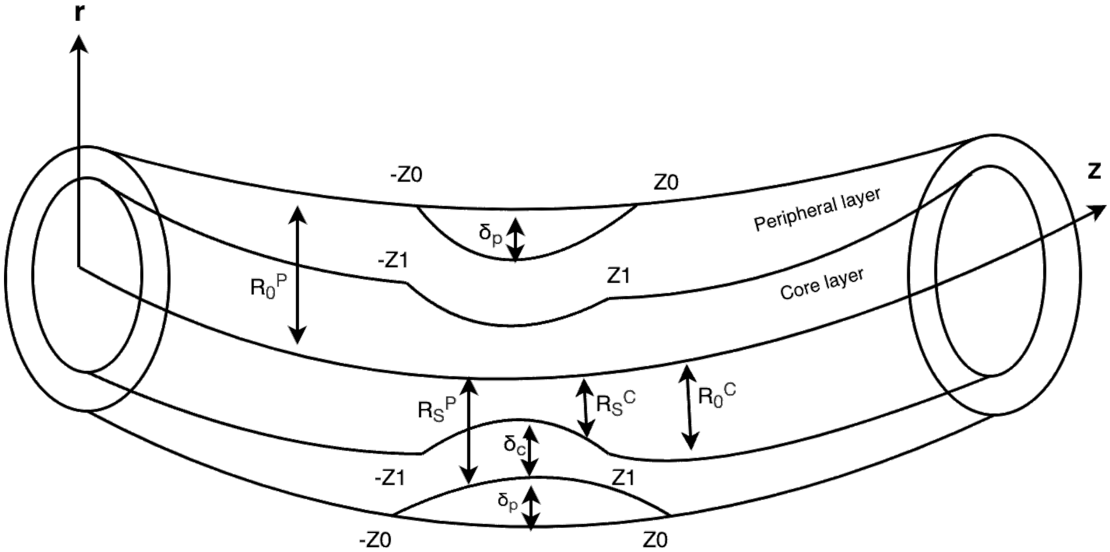

Fig. 1: Sketch of two-layer blood flow in curved stenosed artery.

On integration:

$$-\frac{Pr^2}{2\mu_p} + A(z) = r\frac{\partial v_p}{\partial r} + \frac{\kappa P}{16\mu_p}\Big((R_s^p)^2 - r^2\Big)^2.$$

Using boundary conditions $\frac{\partial v_p}{\partial r} = 0$ at $r = 0$, we get:

$$\frac{\partial v_p}{\partial r} = -\frac{Pr}{2\mu_p} + \frac{\kappa P(R_s^p)^2 r}{8\mu_p} - \frac{\kappa P r^3}{16\mu_p}.$$

Again integration:

$$v_p = -\frac{Pr^2}{4\mu_p} + \frac{\kappa P(R_s^p)^2 r^2}{16\mu_p} - \frac{\kappa P r^4}{64\mu_p} + B(z).$$

After simplification and using boundary conditions $v_p = 0$ at $r = R_s^p$, we receive:

$$B(z) = \frac{P(R_s^p)^2}{4\mu_p} - \frac{3\kappa P(R_s^p)^4}{64\mu_p}.$$

Substituting the value of $B(z)$:

$$\begin{aligned} v_p &= \frac{P}{4\mu_p}\Big((R_s^p)^2 - r^2\Big) \\ &\quad + \frac{\kappa P}{64\mu_p}\Big(4r^2(R_s^p)^2 - 3(R_s^p)^4 - r^4\Big). \end{aligned} \tag{12}$$

2) Core layered velocity: Let v_c be the core layered velocity, i.e., region $0 \le r \le R_s^c$. Then, from equation (11):

$$-P\frac{r}{\mu_c} = \frac{\partial}{\partial r}\Big(r\frac{\partial v_c}{\partial r} + \frac{\kappa P}{16\mu_c}\Big((R_s^c)^2 - r^2\Big)^2\Big).$$

On integration:

$$-\frac{Pr^2}{2\mu_c} + C(z) = r\frac{\partial v_c}{\partial r} + \frac{\kappa P}{16\mu_c}\Big((R_s^c)^2 - r^2\Big)^2.$$

Using boundary conditions $\frac{\partial v_c}{\partial r} = 0$ at $r = 0$, then it becomes:

$$\frac{\partial v_c}{\partial r} = -\frac{Pr}{2\mu_c} + \frac{\kappa P(R_s^c)^2 r}{8\mu_c} - \frac{\kappa P r^3}{16\mu_c}.$$

Again integration:

$$v_c = -\frac{Pr^2}{4\mu_c} + \frac{\kappa P(R_s^c)^2 r^2}{16\mu_c} - \frac{\kappa P r^4}{64\mu_c} + D(z).$$

Using boundary conditions $v_c = v_p$ at $r = R_s^c$, then:

$$\begin{aligned} D(z) &= \frac{P}{4\mu_p}\Big((R_s^p)^2 - r^2\Big) + \frac{P(R_s^c)^2}{4\mu_c} - \frac{3\kappa P(R_s^c)^4}{64\mu_c} \\ &\quad + \frac{\kappa P}{64\mu_p}\Big(4r^2(R_s^p)^2 - 3(R_s^p)^4 - r^4\Big). \end{aligned}$$

After substitution of the value of $D(z)$ and simplification, the core-layer velocity becomes:

$$\begin{aligned} v_c &= \frac{P}{4\mu_c}\Big(\bar{\mu}(R_s^p)^2 + (R_s^c)^2 - (1+\bar{\mu})r^2 \\ &\quad + \frac{\kappa}{16}\Big(4r^2\big((R_s^c)^2 + \bar{\mu}(R_s^p)^2\big) \\ &\quad - 3\big(\bar{\mu}(R_s^p)^4 + (R_s^c)^4\big) - (1+\bar{\mu})r^4\Big)\Big), \end{aligned} \tag{13}$$

where $\bar{\mu} = \frac{\mu_c}{\mu_p}$.

B. Two-layered volumetric flow rate in curved artery

1) Peripheral layer: Let us consider volumetric flow rate in peripheral layer to be Q_p [26, 27]:

$$Q_p = \int_{R_s^c}^{R_s^p} 2\pi r v_p dr.$$

Substituting v_p from (12), then integration, and after simplification:

$$Q_p = \frac{\pi P}{8\mu_p}\Big(\Big((R_s^p)^2 - (R_s^c)^2\Big)^2 + \frac{\kappa}{24}\Big((R_s^c)^6 - 4(R_s^p)^6 + 9(R_s^p)^4(R_s^c)^2 - 6(R_s^p)^2(R_s^c)^4\Big)\Big).$$

Using the geometry of stenosis and then applying the binomial expansion, moreover the maximum height of stenosis is at $z = 0$, and after simplification:

$$\begin{aligned} Q_p = \frac{\pi P(R_0^p)^4}{8\mu_p}\Bigg(&1 - 4\frac{\delta_p}{R_0^p} + \beta^4\Big(1 - 4\frac{\delta_c}{R_0^c}\Big) \\ &- 2\beta^2\Big(1 - 2\frac{\delta_p}{R_0^p} - 2\frac{\delta_c}{R_0^c} + 4\frac{\delta_p\delta_c}{R_0^pR_0^c}\Big) \\ &+ \frac{\kappa(R_0^p)^2}{24}\Bigg(\beta^6\Big(1 - 6\frac{\delta_c}{R_0^c}\Big) - 4\Big(1 - 6\frac{\delta_p}{R_0^p}\Big) \\ &+ 9\beta^2\Big(1 - 2\frac{\delta_c}{R_0^c} - 4\frac{\delta_p}{R_0^p} + 8\frac{\delta_p\delta_c}{R_0^pR_0^c}\Big) \\ &- 6\beta^4\Big(1 - 4\frac{\delta_c}{R_0^c} - 2\frac{\delta_p}{R_0^p} + 8\frac{\delta_p\delta_c}{R_0^pR_0^c}\Big)\Bigg)\Bigg). \end{aligned} \tag{14}$$

2) Core layered volumetric flow rate: Let Q_c be the volumetric flow rate in core layered. Then:

$$Q_c = \int_0^{R_s^c} 2\pi r v_c dr.$$

Substituting v_c from (13), then integration, and after simplification:

$$\begin{aligned} Q_c = \frac{\pi P}{8\mu_c}\Big(&2\bar{\mu}(R_s^p)^2(R_s^c)^2 + 2(R_s^c)^4 - (1+\bar{\mu})(R_s^c)^4 \\ &+ \frac{\kappa}{24}\Big(6(R_s^c)^4\Big((R_s^c)^2 + \bar{\mu}(R_s^p)^2\Big) - (1+\bar{\mu})(R_s^c)^6 \\ &- 9(R_s^c)^2\Big(\bar{\mu}(R_s^p)^4 + (R_s^c)^4\Big)\Big)\Big). \end{aligned}$$

Using the geometry of stenosis and then applying the binomial expansion, moreover the maximum height of stenosis at $z = 0$, we have:

$$\begin{aligned} Q_c = \frac{\pi P(R_0^p)^4}{8\mu_c}\Bigg(&2\bar{\mu}\beta^2\Big(1 - \frac{2\delta_p}{R_0^p} - \frac{2\delta_c}{R_0^c} + \frac{4\delta_p\delta_c}{R_0^pR_0^c}\Big) \\ &+ (1-\bar{\mu})\beta^4\Big(1 - 4\frac{\delta_c}{R_0^c}\Big) \\ &+ \frac{\kappa(R_0^p)^2}{24}\Bigg(6\bar{\mu}\beta^4\Big(1 - 4\frac{\delta_c}{R_0^c} - 2\frac{\delta_p}{R_0^p} + 8\frac{\delta_p\delta_c}{R_0^pR_0^c}\Big) \\ &- (4+\bar{\mu})\beta^6\Big(1 - 6\frac{\delta_c}{R_0^c}\Big) \\ &- 9\bar{\mu}\beta^2\Big(1 - 4\frac{\delta_p}{R_0^p} - 2\frac{\delta_c}{R_0^c} + 8\frac{\delta_p\delta_c}{R_0^pR_0^c}\Big)\Bigg)\Bigg). \end{aligned} \tag{15}$$

C. Two-layered pressure drop in curved artery

1) Pressure drop in peripheral layer: Pressure drop across stenosis in peripheral region is:

$$(\Delta P)_s^p = \int_{-z_0}^{z_0} P dz.$$

We use equation (14):

$$(\Delta P)_s^p = \int_{-z_0}^{z_0} \frac{8\mu_p Q_p dz}{\pi(R_0^p)^4(\dots)},$$

where $(\dots)$ is the biggest bracket from (14).

After integration:

$$(\Delta P)_s^p = \frac{16\mu_p Q_p z_0}{\pi(R_0^p)^4(\dots)}, \tag{16}$$

where $(\dots)$ is again the biggest bracket from (14).

2) Core layer pressure drop: The pressure drop across the stenosis in core region is:

$$(\Delta P)_s^c = \int_{-z_1}^{z_1} P dz.$$

Substituting the value of P from equation (15):

$$(\Delta P)_s^c = \int_{-z_1}^{z_1} \frac{8\mu_c Q_c dz}{\pi(R_0^p)^4(\dots)},$$

where $(\dots)$ is the biggest bracket from (15).

After integration:

$$(\Delta P)_s^c = \frac{16\mu_c Q_c z_1}{\pi(R_0^p)^4(\dots)}, \tag{17}$$

where $(\dots)$ is again the biggest bracket from (15).

D. Two-layered shear stress in curved artery

1) Peripheral layered shear stress: The shear stress on surface of peripheral layer across stenosis at $r = R_s^p$ is given by [27]:

$$\begin{aligned} \tau_s^p &= \Big(-\mu_p\frac{\partial v_p}{\partial r}\Big) \\ &= \Big((-\mu_p)(-P)\frac{r}{2\mu_p}\Big)_{r=R_s^p} = \Big(\frac{PR_s^p}{2}\Big). \end{aligned}$$

Substituting the value of P from equation (14):

$$\tau_s^p = \frac{4\mu_p Q_p R_s^p}{\pi(R_0^p)^4(\dots)},$$

where $(\dots)$ is the biggest bracket from (14).

After simplification:

$$\tau_s^p = \frac{4\mu_p Q_p\Big(1 - \frac{\delta_p}{R_0^p}\Big)}{\pi(R_0^p)^3(\dots)}, \tag{18}$$

where $(\dots)$ is the biggest bracket from (14).

2) Shear stress in core layer: The shear stress in the core layer at $r = R_s^c$ is:

$$\tau_s^c = \Big(-\mu_c \frac{\partial v_c}{\partial r} \Big) = \Big((-\mu_c)(-P) \frac{r}{2\mu_c} \Big)_{r=R_s^c} = \Big(\frac{P R_s^c}{2} \Big).$$

Substituting the value of P from equation (15):

$$\tau_s^c = \frac{4\mu_c Q_c R_s^c}{\pi (R_0^p)^4 (\dots)},$$

where $(\dots)$ is the biggest bracket from (15).

After simplification the final result for shear stress is:

$$\tau_s^c = \frac{4\mu_c Q_c R_0^c \Big(1 - \frac{\delta_c}{R_0^c} \Big)}{\pi (R_0^p)^4 (\dots)}, \tag{19}$$

where $(\dots)$ is the biggest bracket from (15).

III. Results and discussion

In this section, we briefly discuss about velocity profile, volumetric flow rate, pressure drop, and shear stress at peripheral and core-layer in the stenosed region of an artery.

A. Two-layered velocity profile of blood flow through a curved stenotic artery

1) Peripheral-layer velocity profile with variation of curvature and viscosity: Figure 2A shows the peripheral layer velocity distribution with radial distance r for various values of curvature. Curvature κ takes the values $(0.5, 1.0, 1.5, 2.0)$ mm^{-1}s. Radius r in the region of stenosis along the peripheral layer ranges from 0.6 mm to 1.0 mm. The velocity v_p at $r = 0.6$ mm are 9.77 mm s^{-1}, 9.224 mm s^{-1}, 8.674 mm s^{-1}, and 8.125 mm s^{-1}, for the curvature 0.5 mm^{-1} s, 1.0 mm^{-1} s, 1.5 mm^{-1} s, and 2.0 mm^{-1} s respectively. The velocity v_p at $r = 1$ mm are zero for the curvature 0.5 mm^{-1} s, 1.0 mm^{-1} s, 1.5 mm^{-1} s, and 2.0 mm^{-1} s respectively. It is observed that the velocity increases as the curvature decreases.

Figure 2B shows peripheral layer velocity distribution with radial distance r for various values of peripheral layer viscosity. Viscosity coefficient μ_p takes the values $(1.4, 1.5, 1.6, 1.7)$ gram mm^{-1} s^{-1}. Radii in the region of stenosis along the peripheral layer has been taken the value of $R_s^p = 0.6$ mm to 1 mm. The velocity v_p at $r = 0.6$ mm are 12.35 mm s^{-1}, 11.47 mm s^{-1}, 10.71 mm s^{-1}, and 10.04 mm s^{-1}, for the peripheral layer viscosity 1.4 gram mm^{-1} s^{-1}, 1.5 gram mm^{-1} s^{-1}, 1.6 gram mm^{-1} s^{-1}, and 1.7 gram mm^{-1} s^{-1} respectively. It is also observed that velocities are zero at $r = 1$ mm for different values of viscosity.

Figure 2E, describes the distribution of velocity for different values of peripheral viscosity and radius of peripheral artery. Viscosity μ_p takes values $(1.4, 1.5, 1.6, 1.7)$ gram mm^{-1} s^{-1}. Radius of an artery has values $(0.6, 0.7, 0.8, 0.9, 1)$ mm. The velocity v_p at $r = 0.6$ mm and $\mu_{p1} = 1.4$ gram mm^{-1} s^{-1} is 5.433 mm s^{-1}. As the radius increases the velocity decreases for the same viscosity and becomes 0 at $r = 1$ mm. The velocity at $r = 0.6$ mm is 4.482 mm s^{-1} for the viscosity 1.7 gram mm^{-1} s^{-1} and becomes 0 at $r = 1$ mm for the same viscosity.

The velocities at $r = 0.6$ mm are $(5.433, 5.08, 4.762, 4.482)$ mm s^{-1} for the viscosity $(1.4, 1.5, 1.6, 1.7)$ gram mm^{-1} s^{-1} respectively. The velocities are 0 at $r = 1$ mm for the viscosities $(1.4, 1.5, 1.6, 1.7)$ gram mm^{-1} s^{-1} respectively. It is found that the blood velocity gradually diminishes with increasing radius of an artery and viscosity i.e the flow velocity becomes smaller and smaller and finally zero at inner wall of an artery. For equal amount of increases in viscosity and radius of an artery, the velocity at $r = 0.6$ mm has maximum and in the inner wall of an artery has zero.

These results indicate that blood velocity increases with decreasing curvature and viscosity, and the peripheral layer velocity adheres to the no-slip condition. Once more, this demonstrates that a peripheral artery's velocity decreases as its radius and viscosity increase.

2) Core-layer velocity profile with variation of curvature and viscosity: Figure 2C depicts core layer velocity distribution with radial distance r at various values of curvature. Curvature κ takes values $(0.5, 1.0, 1.5, 2.0)$ mm^{-1}s. Radii in the region of stenosis along the core layer has the value $R_s^c = 0.0$ to 0.6 mm. The velocity v_c at $r = 0$ are 14.39 mm s^{-1}, 13.32 mm s^{-1}, 12.25 mm s^{-1}, and 11.17 mm s^{-1}, for the curvature 0.5 mm^{-1} s, 1.0 mm^{-1} s, 1.5 mm^{-1} s, and 2.0 mm^{-1} s respectively. It is observed that the velocity in the core layer decreases with increasing curvature.

The velocity v_c at $r = 0.6$ mm are 6.789 mm s^{-1}, 6.319 mm s^{-1}, 5.85 mm s^{-1}, and 5.38 mm s^{-1} for the curvature 0.5 mm^{-1} s, 1.0 mm^{-1} s, 1.5 mm^{-1} s, and 2.0 mm^{-1} s respectively. Thus the behaviour of blood flow in the core region with the radial co-ordinates for different values of curvature is studied and, it is found that the blood velocity gradually diminishes with increasing r, i.e. the flow velocity becomes smaller and smaller as one proceeds away from the center.

For equal amount of increase in curvature the velocity in the peripheral layer decreases from 9.77 mm s^{-1}

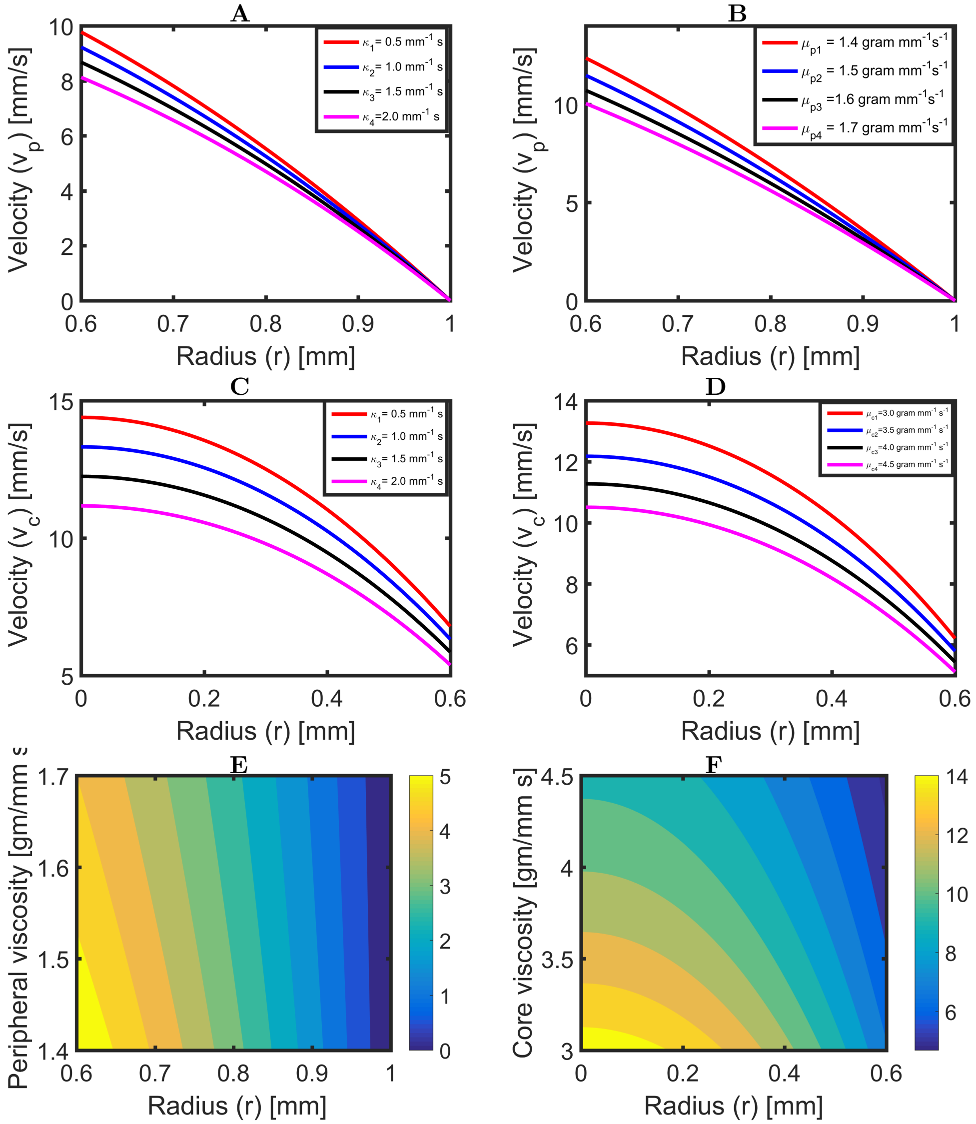

Fig. 2: Variation of peripheral and core layer velocity with radial distance for different curvature (A,C), viscosity (B,D).

to 8.125 mm s^{-1}, but in core layer for equal amount of increase in curvature the velocity decreases from 14.39 mm s^{-1} to 11.17 mm s^{-1}. Curvature takes the values $(0.5, 1.0, 1.5, 2.0)$. The increment velocity in peripheral and core layer are 47.28%, 44.40%, 41%, and 37%, this shows that the effect of curvature is more in peripheral layer velocity.

Figure 2D shows the core velocity distribution with radial distance r for various values of core layer viscosity. Viscosity coefficient μ_c takes values $(3.0, 3.5, 4.0, 4.5)$ gram mm^{-1} s^{-1}. Radii in the region of stenosis along the core layer has been taken the value of $R_s^c = 0.0$ to 0.6 mm. It is observed that velocity distribution in the core layer at $r = 0$ is the maximum at least value of viscosity coefficient μ_{c1} =3.0 gram mm^{-1} s^{-1} which is equal to 13.27 mm s^{-1}, and the velocity is minimum for the viscosity $\mu_{c4} = 4.5$ gram mm^{-1} s^{-1} which is equal to 10.51 mm s^{-1}. when the viscosity coefficient increases uniformly the core velocity decreases. It is also noted that the flow velocity becomes smaller and smaller as one proceed away from the axis regarding its variation with viscosity.

Figure 2F, describes the distribution of velocity for different values of core viscosity and radius of

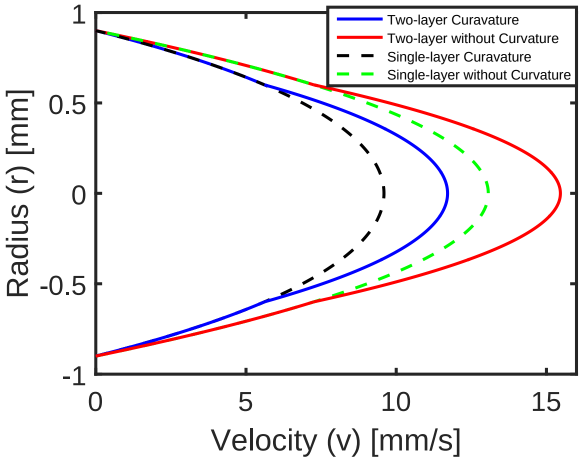

Fig. 3: Single-layer versus two-layer with and without curvature.

Table 1: Single-layer and two-layer velocity with and without curvature.

Velocity/Radius	0.0	0.2	0.6	0.9
1-layer with curvature	9.52	9.147	5.564	0
2-layer with curvature	11.71	11.06	5.615	0
1-layer without curvature	13.06	12.38	7.188	0
2-layer without curvature	15.46	14.55	7.258	0

core artery. Viscosity μ_c takes values $(3.0, 3.5, 4.0, 4.5)$ gram mm^{-1} s^{-1}. Radius of an artery has values $(0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6)$ mm. The velocity v_c at $r = 0.0$ and $\mu_{c1} = 3.0$ gram mm^{-1} s^{-1} is 14.54 mm s^{-1}. As the radius increases the velocity decreases for the same viscosity and becomes 7.031 at $r = 0.6$ mm. The velocity at $r = 0.0$ mm is 9.696 mm s^{-1} for the viscosity 4.5 gram mm^{-1} s^{-1} and becomes 4.687 at $r = 0.6$ mm for the same viscosity.

The velocities at $r = 0.0$ are $(14.54, 12.46, 10.88, 9.696)$ mm s^{-1} for the viscosity $(3.0, 3.5, 4.0, 4.5)$ gram mm^{-1} s^{-1} respectively. The velocities are $(7.031, 6.025, 5.273, 4.687)$ at $r = 0.6$ mm for the viscosities $(3.0, 3.5, 4.0, 4.5)$ gram mm^{-1} s^{-1} respectively.

It is found that the blood velocity diminishes with increasing artery radius and viscosity, reaching minimum at $r = 0.6$ mm. For equal increases in viscosity and radius, the velocity is maximum at $r = 0$ and minimum at $r = 0.6$ mm. Once more, artery's core layer velocity decreases with increasing core viscosity and radius.

3) Comparison of velocity profile between single and two-layer with and without curvature: In table 1, we have examined the velocity profile in both single-layer and two-layer models, considering cases with and without curvature, for various stenosis heights. The results demonstrate that the velocity decreases in both layers as the stenosis height increases, regardless of the presence of curvature. Additionally, the velocity is lower in an artery with curvature compared to one without curvature in both scenarios. This illustrates the impact of curvature and highlights the differences between the velocities in single-layer and two-layer models. Furthermore, the analysis underscores the significance of curvature in the two-layer model. In the second-last column of our results, all velocities are zero at the inner wall of the artery due to the no-slip condition applied in this model.

Figure 3 shows a comparison of velocity profiles in single-layer and two-layer blood flow with and without curvature as radial distance r varies from 0.0 mm to 0.9 mm. We have taken $\kappa = 1.75$ mm^{-1} s, peripheral layer viscosity= 1.55 gram mm^{-1} s^{-1}, core layer viscosity 3.75 gram mm^{-1} s^{-1} and average viscosity for single-layer is 1.55 gram mm^{-1} s^{-1}. At a radial distance of $r = 0$, two-layer velocity with and without curvature are 11.71 mm s^{-1}, 15.46 mm s^{-1} respectively. Similarly in single layer velocity with and without curvature are 9.592 mm s^{-1}, 13.01 mm s^{-1} respectively. In both of these cases it is observed that the velocity is smaller in case of a curved artery. The findings reveal that curvature predominantly influences peripheral layer velocity than the core layer velocity. From this we conclude that the velocity is reduced by curvature and more realistic result can be seen in two-layer curvature model.

B. Two-layered volumetric flow rate through a curved stenotic artery

1) Peripheral-layer volumetric flow rate with variation of curvature and viscosity: Figure 4A depicts the volumetric flow rate (Q_p) of blood in a curved artery across stenosis with height 0 to 0.1 mm. Curvature κ takes the values $(0.5, 1.0, 1.5, 2.0)$ mm^{-1} s, the volumetric flow rate at $\delta_p = 0$ are $(7.31, 6.831, 6.351, 5.872)$ mm^3 s^{-1} respectively. The height of stenosis increases the volumetric flow rate decreasing linearly and becomes closer and closer at $\delta_p = 0.1$ mm for different values of curvature. Figure 4B depicts peripheral layer volumetric flow rate Q_p for different values of viscosity coefficient μ_p.

Viscosity takes the values $(1.4, 1.5, 1.6, 1.7)$ gram mm^{-1} s^{-1}, the volumetric flow rate are 6.766 mm^3 s^{-1},6.315 mm^3 s^{-1}, 5.921 mm^3 s^{-1}, and 5.572 mm^3 s^{-1} respectively at $\delta_p = 0$ mm. The volumetric flow rate are $(2.729, 2.548, 2.388, 2.248)$ mm^3 s^{-1} at $\delta_p = 0.1$ for different values of peripheral layer of viscosity. It concludes that the flow rate decreases with an increase in the viscosity coefficient, and as the height of the stenosis increases, the volumetric flow rate also decreases.

2) Core-layer volumetric flow rate with variation of curvature and viscosity: Figure 4C depicts the volumetric flow rate (Q_c) of blood in an curved artery across stenosis with different heights ranging from 0 to 0.1 mm. Flow rate is minimum for $\kappa_4 = 2.0$ mm^{-1} s and decreases gradually as the height of the stenosis increases and becomes minimum when the thickness becomes 0.1 mm. Volumetric flow rate decreases gradually with increasing curvature and it also decreases linearly as the stenosis thickness increases. When $\kappa_1 = 0.5$ mm^{-1} s, and $\kappa_4 = 2.0$ mm^{-1} s, the corresponding volumetric flow rates are 9.136 mm$^3 s^{-1}$ and 7.845 mm$^3 s^{-1}$ at $\delta_c = 0$ respectively. This figure shows that the volumetric flow rate increases with the decreased curvature. When we measure the values at $\delta_p = 0$, $\delta_c = 0$, in peripheral layer the volumetric flow rate is decreases by 1.438 mm$^3 s^{-1}$, but in the core layer the volumetric flow rate decreases by 1.291 mm$^3 s^{-1}$, this again shows the effect of curvature is more in the peripheral layer.

Figure 4D depicts flux Q_c in the core layer for different values of viscosity coefficient μ_c. When μ_c changes from $(3.0 - 4.5)$ gram mm^{-1} s^{-1}, flux decreases up to 1.906 mm^3 s^{-1}. It concludes that the flux is inversely proportional to the viscosity. Q_c attains maximum value when $\delta_c = 0$ mm which is 9.581 mm^{3}s^{-1} and 7.675 mm^{3}s^{-1} when $\delta_c = 0.1$ mm. The volumetric flow rate decreases rapidly at first and then slowly when the viscosity coefficients are $(3.0 - 4.5)$ gram mm^{-1} s^{-1}. In both layers, the volumetric flow rate decreases with increasing curvature and viscosity.

C. Two-layered pressure drop through a curved stenotic artery

1) Peripheral-layer pressure drop with variation of curvature and viscosity: Figure 5A depicts the peripheral layer pressure drop with height of stenosis δ_p for various values of curvature. Curvature κ takes the values $(0.5, 1.0, 1.5, 2.0)$ mm^{-1}s. Thickness δ_p in the region of stenosis along the peripheral layer ranges from 0.0 to 0.1 mm. The pressure drop $(\Delta P)_s^p$ at $\delta_p = 0$ are 62.38 mm Hg, 66.86 mm Hg, 72.02 mm Hg, and 78.04 mm Hg for the curvature 0.5 mm^{-1} s, 1.0 mm^{-1} s, 1.5 mm^{-1} s, and 2.0 mm^{-1} s respectively. In the stenotic region, we see a uniform change in pressure drop and the line is almost parabolic. Pressure drop changes from 62.38 mm Hg to about 140.10 mm Hg, when the stenosis increases from 0 to 0.1 mm for the curvature $\kappa_1 = 0.5$ mm^{-1} s. It is observed that the pressure drop increases with both increasing curvature and height of stenosis.

Figure 5B depicts the pressure drop in curved arteries with different height of stenosis and for different values of peripheral viscosity μ_p. Viscosity coefficient μ_p takes the values $(1.4, 1.5, 1.6, 1.7)$ gram mm^{-1} s^{-1}. Thickness of stenosis δ_p ranges from 0.0 to 0.1 mm. The pressure drop at $\delta_p = 0$ are 53.01 mm Hg, 60.87 mm Hg, 69.95 mm Hg, and 80.55 mm Hg, for the viscosity $(1.4, 1.5, 1.6, 1.7)$ gram mm^{-1} s^{-1} respectively. The pressure drop increases with the height of stenosis in the curved artery. Additionally, as curvature and viscosity rise, the pressure drop also increases.

Table 2 describes the relationship between increasing stenosis and pressure drop for different values of curvature. Height of the stenosis increases gradually from 0.0 to 0.1 mm and viscosity is kept constant (1.55 gram mm^{-1} s^{-1}). For each value of curvature pressure drop at 0.0 and at 0.1 mm are compared. When curvature is 0.5 mm^{-1}s, the pressure drop increases by 124.59%. As the curvature increases the pressure drop percentage decreases gradually, which is shown in the table. The increment percentage decreases gradually by 10% approximately, and it seems uniform. When curvature increases from 0.5 mm^{-1}s to 2.0 mm^{-1}s, the pressure drop increases by 77.04%, but in case of viscosity the pressure drop is increased by 79.55%, at $\delta_p = 0$. In this case effect of viscosity and curvature are almost equal. Similarly at the lower part of the table shows relationship between increasing stenosis and pressure drop for different values of viscosity. In this case the curvature to keep constant (1.75 mm^{-1}s). For each value of viscosity pressure drop at 0.0 and at 0.1 mm are compared. When the viscosity is 1.4 gram mm^{-1} s^{-1} the percentage increment in pressure drop is 120.37% which decreases gradually by 10% approximately, as the viscosity increases by 0.1 gram mm^{-1} s^{-1}.

2) Core-layer pressure drop with variation of curvature and viscosity: Figure 5C demonstrates pressure drop for different curvature and for different height of stenosis is explained. In this case the pressure drop increases for increasing curvature. Curvature κ takes the values $(0.5, 1.0, 1.5, 2.0)$ mm^{-1}s, the pressure drop at $\delta_c = 0$ are 81.60 mm Hg, 88.08 mm Hg, 95.68 mm Hg, and 104.70 mm Hg, respectively. Thus the pressure drop increases when the curvature and height

Table 2: Peripheral-layer pressure drop with curvature and viscosity for different height of stenosis.

δ_p (mm)	0.0	0.02	0.04	0.06	0.08	0.1	% $(\Delta P)_s^p$	κ
$(\Delta P)_s^p$	62.38	70.21	80.27	93.7	112.5	140.1	124.59	0.5
$(\Delta P)_s^p$	66.86	74.91	85.18	98.71	117.4	144	115.37	1.0
$(\Delta P)_s^p$	72.02	80.3	90.74	104.3	122.6	148.1	105.63	1.5
$(\Delta P)_s^p$	78.04	86.52	97.07	110.5	128.4	152.4	95.28	2.0

δ_p (mm)	0.0	0.02	0.04	0.06	0.08	0.1	% $(\Delta P)_s^p$	μ_p
$(\Delta P)_s^p$	53.01	59.55	67.92	79.03	94.49	116.9	120.37	1.4
$(\Delta P)_s^p$	60.87	68.08	77.23	89.22	105.6	128.8	111.59	1.5
$(\Delta P)_s^p$	69.95	77.85	87.76	100.6	117.7	141.4	102.14	1.6
$(\Delta P)_s^p$	80.55	89.13	99.75	113.9	131	154.7	92.05	1.7

Table 3: Core-layer pressure drop with curvature and viscosity for different height of stenosis.

δ_c (mm)	0.0	0.02	0.04	0.06	0.08	0.1	% $(\Delta P)_s^c$	κ
$(\Delta P)_s^c$	81.86	85.1	88.92	93.1	97.69	102.7	25.45	0.5
$(\Delta P)_s^c$	88.08	91.87	96.01	100.5	105.5	110.9	25.90	1.0
$(\Delta P)_s^c$	95.68	99.82	104.3	109.3	114.7	120.6	26.04	1.5
$(\Delta P)_s^c$	104.7	109.3	114.2	119.7	125.7	132.1	26.17	2.0

δ_c (mm)	0.0	0.02	0.04	0.06	0.08	0.1	% $(\Delta P)_s^c$	μ_c
$(\Delta P)_s^c$	66.64	70.64	75.15	80.27	86.14	92.79	39.24	3.0
$(\Delta P)_s^c$	82.34	88.89	91.97	97.69	104.2	111.4	35.29	3.5
$(\Delta P)_s^c$	101.4	106.5	112.1	118.3	125.3	132.8	30.96	4.0
$(\Delta P)_s^c$	124.8	130.2	136.1	142.6	149.8	157.5	26.20	4.5

Table 4: Peripheral-layer shear stress with curvature and viscosity for different height of stenosis.

δ_p (mm)	0.0	0.02	0.04	0.06	0.08	0.1	% (τ_s^p)	κ
$(\tau)_s^p$	31.74	35.11	39.5	45.46	54.01	66.96	110.96	0.5
$(\tau)_s^p$	33.8	37.22	41.63	47.53	55.84	68.07	101.39	1.0
$(\tau)_s^p$	36.15	39.6	44	49.80	57.79	69.21	91.45	1.5
$(\tau)_s^p$	38.85	42.31	46.66	52.3	59.88	70.39	81.18	2.0

δ_p (mm)	0.0	0.02	0.04	0.06	0.08	0.1	%(τ_s^p)	μ_p
$(\tau)_s^p$	38.94	42.85	47.87	54.57	63.97	77.73	99.61	1.4
$(\tau)_s^p$	41.73	45.91	51.29	58.47	68.54	83.29	99.59	1.5
$(\tau)_s^p$	44.51	48.97	54.71	62.37	73.11	88.84	99.59	1.6
$(\tau)_s^p$	47.29	52.03	58.13	66.27	77.67	94.39	99.59	1.7

Table 5: Core-layer shear stress with curvature and viscosity for different height of stenosis.

δ_c (mm)	0.0	0.04	0.08	0.12	0.16	%(τ_s^c)	κ
$(\tau)_s^c$	40.24	44.74	51.26	61.74	81.41	102.31	0.5
$(\tau)_s^c$	41.63	46.33	53.14	64.38	84.91	103.96	1.0
$(\tau)_s^c$	43.13	48.04	55.18	66.75	88.72	105.70	1.5
$(\tau)_s^c$	44.73	49.87	57.37	69.57	92.89	107.66	2.0

δ_c (mm)	0.0	0.04	0.08	0.12	0.16	% (τ_s^c)	μ_c
$(\tau)_s^c$	36.18	40.75	47.58	59.13	82.92	129.11	3.0
$(\tau)_s^c$	41.27	46.12	53.22	64.86	87.47	111.94	3.5
$(\tau)_s^c$	46.26	51.36	58.7	70.44	92.24	99.39	4.0
$(\tau)_s^c$	52.05	57.78	66.04	79.25	103.8	99.42	4.5

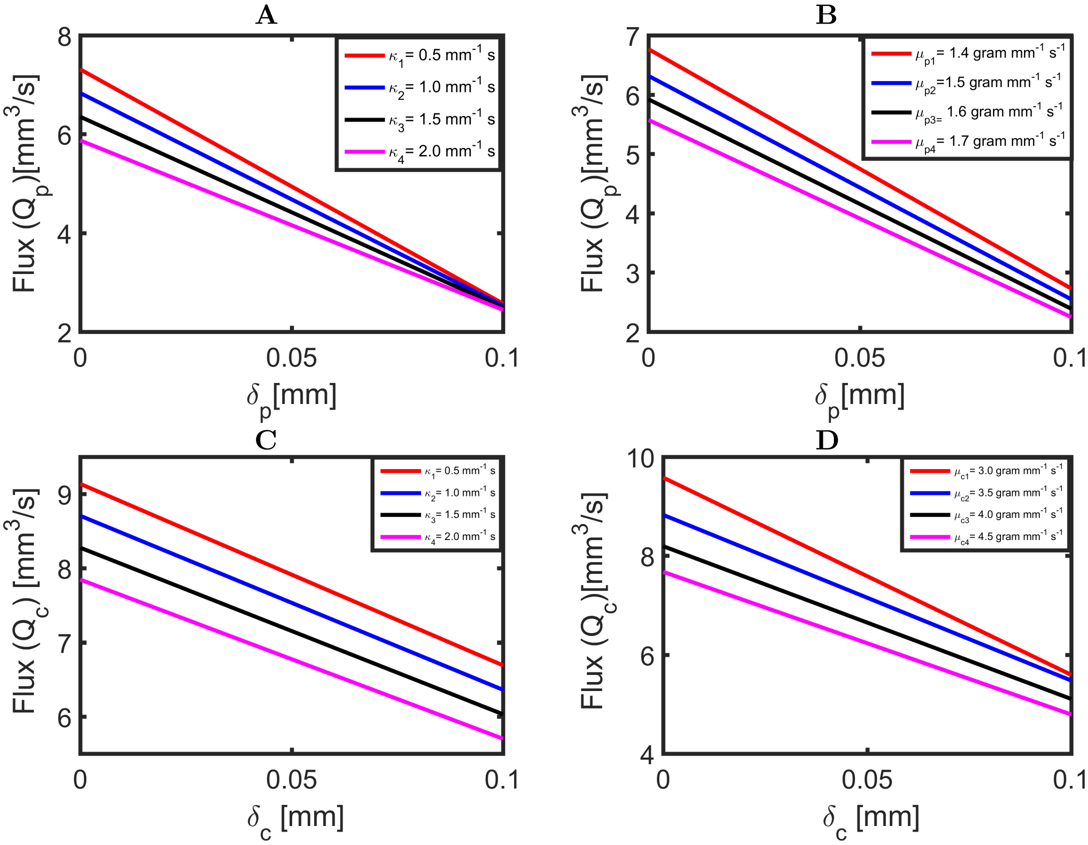

Fig. 4: Variation of peripheral and core layer volumetric flow rate with thickness of stenosis for different curvature (A,C), viscosity (B,D).

of the stenosis increases. Figure 5D depicts the blood pressure drop in curved arteries with different heights of stenosis and for different values of core viscosity μ_c. Core viscosity μ_c takes $(3.0, 3.5, 4.0, 4.5)$ gram mm^{-1} s^{-1}, the pressure drops at $\delta_c = 0$ are 66.64 mm Hg, 82.34 mm Hg, 101.40 mm Hg, and 124.8 mm Hg respectively. It observed that pressure drops increases for increasing core viscosity.

Table 3 describes the effect of increasing stenosis and curvature upon the pressure drop. Thickness of the stenosis increases gradually from 0.0 to 0.1 mm. For each value of curvature, pressure drop at 0.0 and 0.1 mm are compared with their percentage increment. Pressure drop increases about 25.45% approximately for the increment of 0.5 mm^{-1}s curvature. But this increasing percentage increases gradually due to increasing curvature. The ratio of increment is almost uniform.

Similarly in the lower part of the table shows relationship between increasing stenosis and pressure drop for different values of viscosity. In this case the curvature is kept constant (1.75 mm^{-1}s). For each value of core viscosity pressure drop at 0.0 and at 0.1 mm are compared. When the viscosity is 3.0 gram mm^{-1} s^{-1} the percentage increment in pressure drop is 39.24% which decreases gradually by 5% approximately, as the viscosity increases by 0.5 gram mm^{-1} s^{-1}.

Pressure drops increase with heightened curvature and viscosity, particularly impacting stenosis height in the peripheral layer.

D. Two-layered shear stress through a curved stenotic artery

1) Peripheral-layer shear stress with variation of curvature and viscosity: Figure 6A depicts the shear stress under various conditions of curvature and height of stenosis. Curvature κ takes the values $(0.5, 1.0, 1.5, 2.0)$ mm^{-1} s. Thickness of stenosis along peripheral layer ranges from 0.0 to 0.1 mm. The shear stress at $\delta_p = 0$ are 31.74 gram mm^{-1} s^{-2}, 33.80 gram

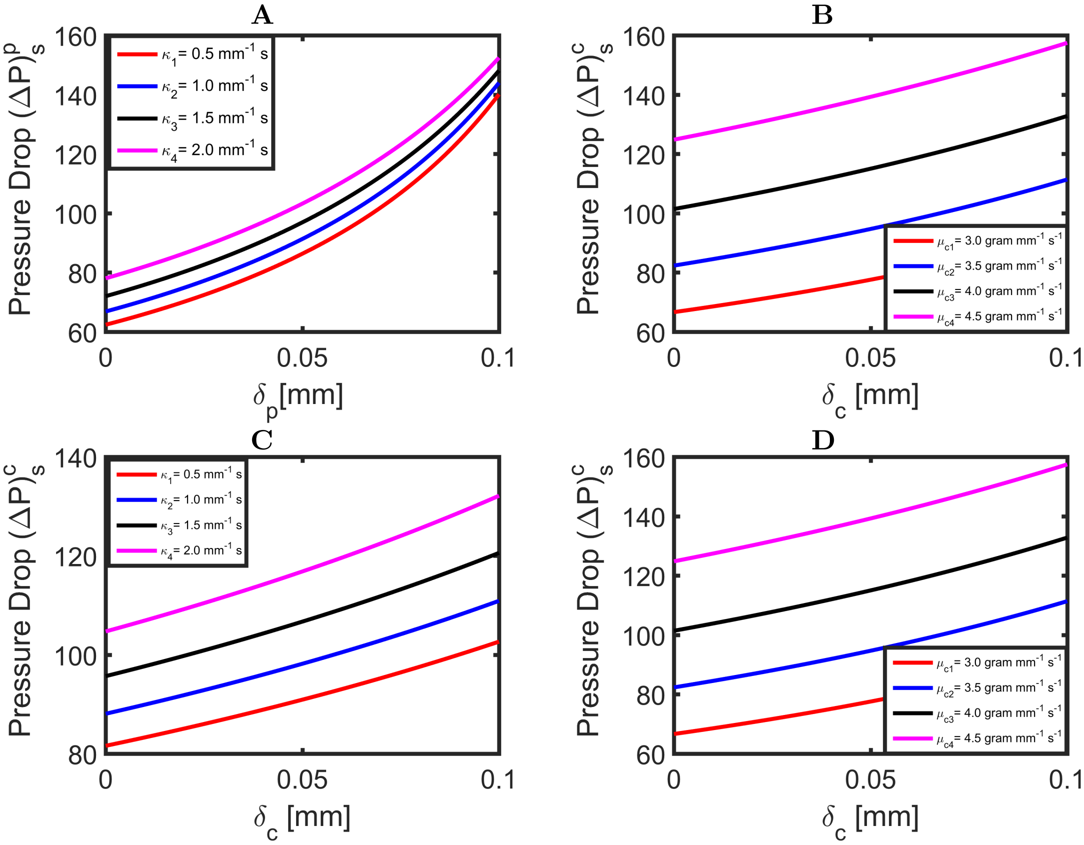

Fig. 5: Variation of peripheral and core layer pressure drop with thickness of stenosis for different curvature (A,C), viscosity (B,D).

mm^{-1} s^{-2}, 36.15 gram mm^{-1} s^{-2}, and 38.85 gram mm^{-1} s^{-2}, for the curvature $(0.5, 1.0, 1.5, 2.0)$ mm^{-1} s respectively. This shows that shear stress increases with increasing curvature. It is also noted that the shear stress increases gradually with an increase in stenosis height, This figure tells us that the shear stress is affected by stenosis height and curvature.

Figure 6B depicts the shear stress in curved arteries with different height of stenosis and for different values of peripheral viscosity μ_p. For $\mu_{p1} = 1.4$ gram mm^{-1} s^{-1}, the shear stress is 38.94 gram mm^{-1} s^{-2} at $\delta_p = 0$, and for $\mu_{p4} = 1.7$ gram mm^{-1} s^{-1}, the shear stress is nearly 47.29 gram mm^{-1} s^{-2} at $\delta_p = 0$. As height of the stenosis increases the shear stress increases gradually. When $\delta_p = 0.0$, the shear stress are $(38.94, 41.73, 44.51, 47.29)$ gram mm^{-1} s^{-2}, for the viscosity $(1.4, 1.5, 1.6, 1.7)$ gram mm^{-1} s^{-1}. It means that when blood reaches the stenosis region initially, the shear stress is normal as before without the stenosis region.

Table 4 viscosity is kept constant and curvature is increased gradually and it is shown that the shear stress is increasing for increasing height of stenosis and curvature. The curvature are increased by 0.5 mm^{-1}s in each step. For this equal increment of curvature, corresponding shear stress is shown in the table. Again for each value of curvature shear stress at 0.0 and at 0.1 mm are compared, when curvature is 0.5 mm^{-1}s the shear stress increases by 110.95%. As the curvature increases the shear stress percentage decreases gradually, which is shown in the table. The increment percentage decreases gradually by 10% approximately, and it seems uniformly.

Similarly at the lower part of the table shows relationship between increasing stenosis and shear stress for different values of viscosity. In this case the curvature to keep constant (1.75 mm^{-1}s). For each value of viscosity pressure drop at 0.0 and at 0.1 mm are compared. When the viscosity is 1.4 gram mm^{-1} s^{-1} the percentage increment in pressure drop is 99.61% which

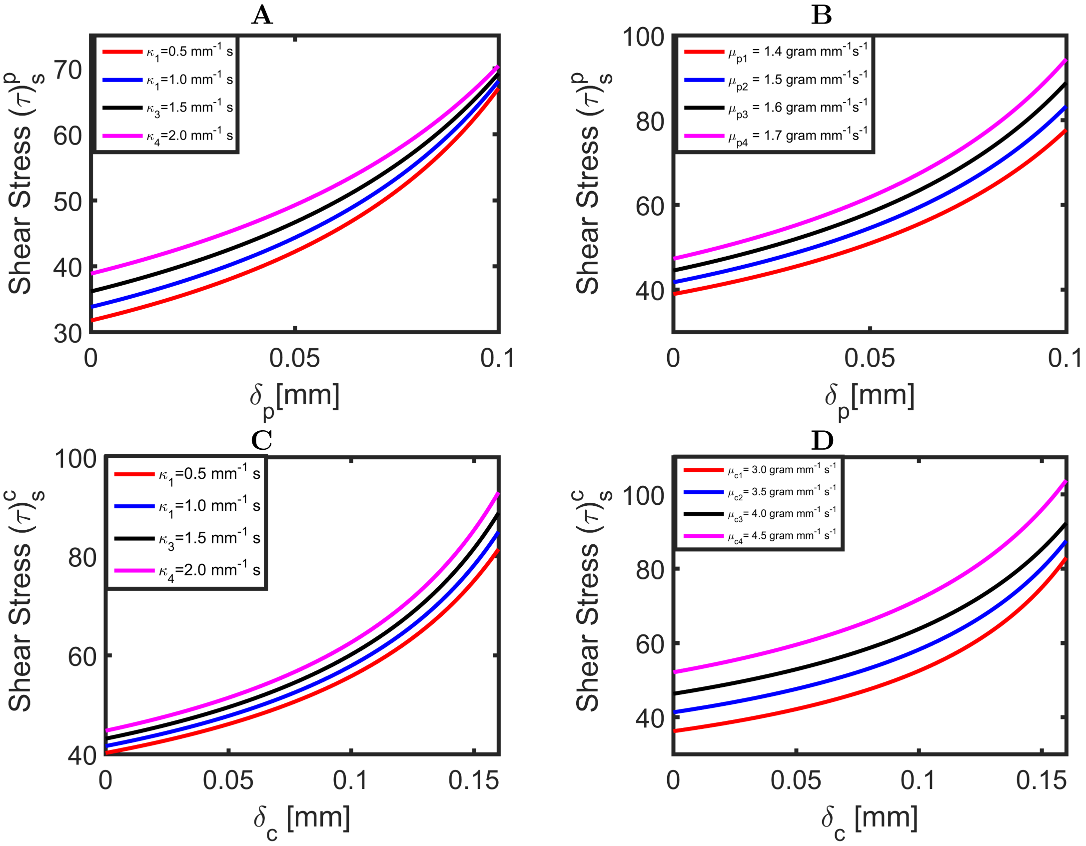

Fig. 6: Variation of peripheral and core Layer shear stress with thickness of stenosis for different curvature (A,C), viscosity (B,D).

decreases gradually by less than 1% approximately, as the viscosity increases by 0.5 gram mm^{-1} s^{-1}.

2) Core-layer shear stress with variation of curvature and viscosity: Figure 6C shows how blood flow shear stress in a curved artery with different height of stenosis. In this figure shear stress for different values of curvature and for different height of stenosis is explained. Curvature κ takes the values $(0.5, 1.0, 1.5, 2.0)$ mm^{-1}s. Thickness δ_c ranges from 0.0 to 0.15 mm. The shear stress τ_s^c at $\delta_c = 0$ are 40.24 gram mm^{-1} s^{-2}, 41.63 gram mm^{-1} s^{-2}, 43.13 gram mm^{-1} s^{-2}, 44.73 gram mm^{-1} s^{-2}, for the curvature $(0.5, 1.0, 1.5, 2.0)$ mm^{-1} s respectively. This shows that in a curved artery having both curvature and stenosis.

The shear stress increases for increasing stenosis and curvature. For $\kappa = 0.5$ mm^{-1}s, the shear stress is equal to 81.41 gram mm^{-1} s^{-2} when $\delta_c = 0.16$ mm. For $\kappa = 2.0$ mm^{-1} s, shear stress is equal to 92.89 gram mm^{-1} s^{-2} when $\delta_c = 0.16$ mm. As the height of the stenosis increases shear stress increases gradually.

Figure 6D depicts the blood shear stress in curved artery with different height of stenosis and for different values of core viscosity μ_c. Viscosity takes the values μ_c $(3.0, 3.5, 4.0, 4.5)$ gram mm^{-1} s^{-1}. Thickness δ_c along core layer ranges 0.0 to 0.16 mm. The shear stress at $\delta_c = 0$ are 36.18 gram mm^{-1} s^{-2}, 41.27 gram mm^{-1} s^{-2}, 46.26 gram mm^{-1} s^{-2}, 52.05 gram mm^{-1} s^{-2}, for the core viscosity μ_c $(3.0, 3.5, 4.0, 4.5)$ gram mm^{-1} s^{-1} respectively.

As thickness of the stenosis increases shear stress increases gradually. At $\delta_c = 0.16$ mm shear stress is 103.80 gram mm^{-1} s^{-2}, when viscosity is $\mu_{c4} = 4.5$ gram mm^{-1} s^{-1}. It is observed that the shear stress increases with increasing core viscosity and thickness of stenosis.

Table 5 describes the effect of increasing stenosis and curvature upon the shear stress. Thickness of the stenosis increases gradually from 0.0 to 0.16 mm. For each value of curvature,shear stress at 0.0 and 0.16 mm are compared with their percentage increment. Shear stress increases about 102.31% approximately for 0.5 mm^{-1}s curvature value. But this increasing percentage increases gradually due to increasing curvature. The ratio of increment is almost uniform.

Similarly in the lower part of the table shows relationship between increasing stenosis and shear stress for different values of viscosity. In this case the curvature is kept constant (1.75 mm^{-1}s). For each value of core viscosity shear stress at 0.0 and at 0.1 mm are compared. When the viscosity is 3.0 gram mm^{-1} s^{-1} the percentage increment in shear stress is 129.11% which decreases gradually by $(17.17, 12.5)\%$ and then increases by 0.03% approximately, as the viscosity increases by 0.5 gram mm^{-1} s^{-1}.

Shear stresses exhibit an increase with elevated curvature and viscosity, notably affecting stenosis height in the peripheral layer as compare to core layer. The analysis underscores the significance of understanding blood flow mechanisms in a two-layered model, offering valuable insights for diagnosing and treating arterial stenosis. This information has potential applications for biomedical engineers and medical doctors in the diagnosis of cardiovascular diseases.

IV. Conclusion

This article analyzes steady, laminar, and axisymmetric flow in a curved artery, considering the inner core layer of red blood cells and the outer peripheral plasma layer. The model for blood flow in a mildly stenosed-curved artery has been extended by including an axial curvature term. Mathematical expressions for a two-layered velocity profile, volumetric flow rate, pressure drop, and shear stress are analytically evaluated.

The velocity profile, volumetric flow rate, pressure drop, and shear stress in the core and peripheral layers with different values of curvature and viscosity coefficients are considered for result analysis. Maximum velocity is attended at the center of the artery, and minimum at the inner wall. In both layers, the velocity decreases as the curvature and the coefficient of viscosity increases. Curvature primarily impacts peripheral layer velocity, while viscosity has a greater effect on the peripheral layer compared to the core layer, collectively influencing changes in velocity.

A different phenomenon can be seen in case of volumetric flow rate, the flow rate declines with the rise of stenotic height. As viscosity and curvature increase in both layers, the volumetric flow rate experiences a notable decrease, highlighting the greater influence of curvature on the peripheral layer. Moreover, in both the peripheral and core layers, the pressure drop increases with heightened curvature and viscosity, signifying a more pronounced impact on stenosis height in the peripheral layer than in the core layer. As the coefficient of viscosity and curvature increase in the peripheral and core layer, shear stress also increases.

In the peripheral layer, curvature has a more significant impact on shear stress compared to peripheral viscosity, while in the core layer, core viscosity exerts a greater influence than curvature. Additionally, an increase in the height of stenosis leads to heightened pressure drop and shear stress in both the peripheral and core layers, indicating a correlation between stenosis height and these flow parameters.

The findings of the current analysis underscore the critical importance of comprehending and applying the intricacies of blood flow dynamics within a two-layered model. Moreover, these results furnish pivotal insights that can significantly advance the diagnosis and treatment strategies for arterial stenosis. Furthermore, these insights hold promising potential for biomedical engineers, offering valuable tools for enhancing the diagnostic capabilities crucial to medical practitioners in the realm of cardiovascular disease.

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Author contribution statement

Jeevan Kafle: Conceived the concept, planned the study, and revised the manuscript. Chudamani Pokharel: Contributed to the analytical solution, interpreted the results, and initiated the drafting of the manuscript. Pushpa N. Gautam: Assisted in the analytical solution and analysis of the results. Chet R. Bhatta: Edited the manuscript and monitored the study.

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