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Biomath Forum
ORIGINAL ARTICLE

Effect of curvature on hemodynamics in a stenotic artery

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Received: 6 September 2024, Published: 17 January 2025

Abstract: **The formation of stenosis in the lumen obstructs the normal flow and causes disorders in the cardiovascular system, which becomes riskier due to the curvature of the artery. The curvature affects the blood flow system directly by reducing the velocity, which helps increase the thickness of the stenosis. In this article, the effect of curvature on the stenosed part of an artery is studied using the Navier-Stokes equation in cylindrical polar form. A new model is developed by incorporating a term in quadratic form to address the joint effect of stenosis and curvature upon flow parameters. This model equation with appropriate boundary conditions is solved to get analytical solutions for velocity, volumetric flow rate, pressure drop ratio, and shear stress ratio, and all the results are analyzed geometrically. The results show that when the curvature increases, the pressure drop ratio and shear stress ratio increase, while the velocity and volumetric flow rate decrease. Quantitative effect of the curvature on flow parameters is visualized which may help researchers in the related field.**

Keywords: **mild stenosis, arterial lumen, curvature, flow parameters, shear stress**

I. Introduction

Blood contains liquid plasma and suspended erythrocytes, which carry oxygen and digested food particles to every cell of our body with the help of arteries [1]. Stenosis is the hard coating in an artery's inner wall due to the deposition of cholesterol or other fatty particles which reduces the lumen [2]. Blood flow restriction occurs when the lumen narrows as a result of stenosis. Stenosis mostly occurs on the side branches at the bifurcation and in the inner wall of the curvature [3]. Stenosis is one of the major causes of cardiovascular disease which claims a large number of deaths every year. In 2019, 38% of the 17 million premature deaths caused by noncommunicable diseases were due to cardiovascular related disease [4]. Cerebral strokes, destruction of the cardiovascular system, heart failure, and paralysis by obstructing the blood flow to the brain are major effects of stenosis [5].

The blood pressure rises to sustain the blood supply as the stenosis increases. High pressure and narrow artery provokes to increase in the velocity and shear stress [6]. Even severe constriction, which blocks about 90% of the lumen remains unnoticed, so it is difficult to be notified previously by medical symptoms [7]. This indicates the importance of the study on blood flow and its modeling. At a low shear rate of about $0.1\ \mathrm{s}^{-1}$, blood becomes more viscous, reaching a viscosity of approximately 60 cP and this situation is favorable for the deposition [8]. Fibrinogen plays a crucial role in the presence of yield stress in human blood [9]. Blood

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Citation: Pushpa Nidhi Gautam, Jeevan Kafle, Effect of curvature on hemodynamics in a stenotic artery, Biomath 14 (2025), 2501175, https://doi.org/10.55630/j.biomath.2025.01.175

exhibits non-Newtonian behavior for a sufficiently low shear rate and behaves nearly Newtonian for a shear rate greater than 200 s^{-1}, where the viscosity remains nearly constant [10]. The main variables influencing the flow parameters are the artery's curvature, the amount and height of stenosis, the proportion of hematocrit in the blood, and the deformability of red blood cells [10,11].

Young [12] initiated a mathematical study of blood flow by using magnitude analysis. Lee and Fung [13] have studied the flow in a locally constricted tube at a low Reynolds number and concluded that the maximum shear stress occurs near the narrowest cross-section. The motion in a curved tube is not parallel and uniform, as a transverse velocity component emerges in the curved sections, which makes the calculation of flow dynamics complicated [14].

Pao et al. [15] have calculated the twisting rate per unit length and change of curvature and torsion at bifurcation. The authors have also mentioned different formulas to calculate the curvature. When comparing static and dynamic curvature tube models, the dynamic curvature has a more significant impact on increasing the mean wall shear rate than the static curvature [16].

Schilt et al. [17] have studied the impact of time-varying curvature on the velocity profile and concluded that the curvature caused the velocity to be biased towards the outer wall, which depends upon the instantaneous movement of the tube.

Creating Artero-Venous Fistulae (AVF) by grafting a vein onto the outer curvature of a curved artery induces regions of pathologically low wall shear stress, while the inner curvature reduces the arterial area exposed to this low shear stress [18]. Coronary arteries deform with the heart's surface during myocardial contraction. Due to this curvature varies upto 80% of the radius of the curvature [15]. The flow in a coronary artery is unique because the curvature changes with time [17].

Das et al. [19] studied the blood flow characteristics in small arteries with curvature and stenosis when catheters of different sizes were inserted and concluded that as catheter size increases so does the magnitude of pressure drop and shear stress. In this work it is concluded that the secondary stream line divides above and below the cross-sectional plain.

Nosovitsky et al. [20] have explained that the left circumflex coronary artery with high degree curvature has a huge impact on velocity and wall shear stress when the flow is laminar.

Yao et al. [21] have studied the combined effect of different angles of curvature, different degrees of stenosis, and varying Reynolds numbers. The authors have concluded that the secondary flow, skewing of velocity, and wall shear stress are high in the curved portion of an artery.

Gautam et al. [22] have considered blood as a non-Newtonian to study the effect of increasing stenosis upon velocity, volumetric flow rate, shear stress and pressure drop. Kafle et al. [23] have studied the effect of curvature on hemodynamics and analyzed the results. Pokharel et al. [24] have calculated the flow parameters in a stenosed artery taking blood as a non-Newtonian.

Dean [25] has studied the effect of curvature in the fluid flow in a tube and shown a detailed picture of the fluid motion above and below the central plane of the curved tube. According to Dean [26] the reduction in flow due to curvature can be described by a single variable K, defined as the product of the square of the Reynolds number and the ratio of the tube's diameter to the radius of curvature of the coiled tube. Collins and Dennis [27] have used a parameter D similar to K to study the motion of viscous fluid in a coiled circular tube.

Kumawat et al. [28] have studied two-phase flow in a stenosed curved artery taking different viscosity for the core and peripheral layer and analyzing the effect of the curvature parameter, height of the stenosis, hematocrit, magnetic field and peripheral viscosity on flow parameters.

Padmanavan and Jayaraman [29] have studied the effect of curvature and stenosis on flow parameters by using the perturbation method. The authors have mentioned that the secondary flow is induced in a constricted curved tube and secondary flow is different at every curvature point of an artery.

According to Prosi et al. [30] the velocity profiles are skewed highly near the inner wall of the bifurcation which results low shear stress at the outer wall and high shear stress at the flow divider wall and the low shear stress area is supposed to be suitable for the deposition.

Kolandavel et al. [31] have studied the effect of wall motion on mass transport in the left anterior descending coronary artery. In this work, the authors have concluded that the velocity skews and the wall shear stress increases towards the outer wall of the maximum curvature region. Arterial curvature has a huge impact on blood flow and oxygen transport in Artero-Venous Fistulae (AVF) [32].

Iori et al. [18] have suggested that to avoid intimal hyperplasia (IH) straight artery is to be made in AVF using a vein graft.

Ku [33] has used the Womersley number, a ratio of viscous force to the unsteady inertial force and he concluded that the viscous force dominates when Womersley parameter is low and the unsteady inertial force dominates for Womersley parameter greater 10. The initial introduction of curvature has a more significant effect on hemodynamics than further changes, with the left anterior descending artery being most affected by these curvature variations [34]. Variation in curvature in different branches with fixed bifurcation is also shown by the authors in this article.

Yang et al. [35] have discussed in detail, how coronary tortuosity is affected by hypertension, smoking, diabetes, heart failure, and gender. The authors have concluded that the branches of the Left anterior descending (LAD) artery have a large impact on the curvature in low time-averaged wall shear stress (TAWSS) and at high relative residence time (RRT).

In the above mentioned literature review most of the authors support incorporating the curvature term to address the reduction in velocity due to curvature. To account for the effects of curvature, we have incorporated a quadratic curvature term $\lambda\kappa v^2$ which is derived from the principle of centrifugal force, acting outwards in the curved section of the artery upon the fluid particles. Simply the centrifugal force is $\frac{mv^2}{r}$ where, v is the velocity of the fluid, r radius of the curvature and m is the mass of the moving particle.

For a particle of unit mass, the force becomes $\frac{v^2}{r}$, which is equivalent to κv^2 as we have used $\frac{1}{r} = \kappa$. The force κv^2 acts continuously till the mass travels the curved part of the artery. So the quantity κv^2 is multiplied by the time factor λ to get the curvature term $\lambda\kappa v^2$.

The unit of this quantity is same as the velocity so this quantity is added in the velocity term of the z-component of the Navier-Stokes equation, then this new model after incorporating the curvature term is solved for analytical solutions of the flow parameters. The results are fitted rightly to the existing values. After solving the equation, analytical solutions for the velocity profile, volumetric flow rate, shear stress ratio, and pressure drop ratio are obtained in a curved artery with mild stenosis and these solutions are analyzed by using computer software.

II. Extended blood flow curvature model in quadratic form

In this mathematical model, it is assumed that the flow is laminar, axi-symmetric and fully developed in the stenosed region of a curved artery. Cylindrical polar coordinate system of Navier-Stokes equation is used with appropriate boundary and initial conditions. The radius of the stenosed artery is R, and R_0, in non-stenosed region, and r varies from 0 to R. Out of three velocity components v^r, v^θ and v^z along r, θ and z direction respectively. Since the flow is steady, fully developed, and only in z direction with pressure gradient $\dfrac{\partial p}{\partial z}$, the z component of the Navier-Stokes equation after incorporating the quadratic curvature term reduces to

$$\frac{\partial p}{\partial z} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r} + \lambda\kappa v^2\right), \tag{1}$$

where $v = v^z$, and this equation (1) is used to calculate the flow parameters.

A. Geometry of stenosis

Geometry of stenosis is defined as [36]:

$$\frac{R}{R_0} = \begin{cases} 1 - \frac{\delta}{2R_0}\left(1 + cos\frac{\pi z}{z_0}\right), & \text{for } |z| \le z_0, \\ 1, & \text{for } |z| > z_0. \end{cases} \tag{2}$$

Here R and R_0 are the radii in the stenosed and non-stenosed region.

B. Velocity profile in a curved artery

As we have the velocity in a normal straight artery is [36],

$$v = \frac{P}{4\mu}(R^2 - r^2), \tag{3}$$

where the pressure $P(z) = -\dfrac{\partial p}{\partial z}$, R is radius of the stenosed artery and r varies from 0 to R. This velocity in equation (3) is used to construct the curvature term $\lambda\kappa v^2$ and the velocity in the curved part of the artery is denoted by v_c Using the above mentioned conditions, the equation (1) changed to,

$$-P\frac{r}{\mu} = \frac{\partial}{\partial r}\left(r\frac{\partial v_c}{\partial r} + \lambda\kappa v^2\right), \tag{4}$$

with boundary conditions,

$$v = \begin{cases} 0, & \text{at } |z| \le z_0, \quad \text{for} \quad r = R(z), \\ 0, & \text{at } |z| > z_0, \quad \text{for} \quad r = R_0, \end{cases}$$

and

$$\frac{\partial v}{\partial r} = 0, \quad \text{at} \quad r = 0.$$

It is mentioned earlier that the study is focused on the stenosed region of a curved artery, where the radius decreases from R_0 to R. To address this situation, we have incorporate the curvature term $\lambda\kappa v^2$ with v given in equation (3) and the velocity in curved part v_c is

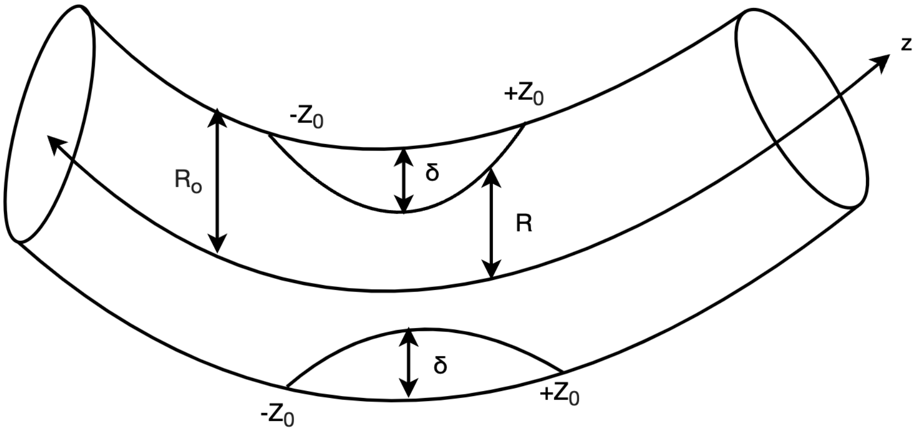


Fig. 1: Part of the curved artery with stenosis.

calculate by integrating equation (4). First equation (4) is integrated using boundary conditions $\frac{\partial v}{\partial r} = 0$ at $r = 0$

$$\frac{\partial v_c}{\partial r} = -\frac{Pr}{2\mu} + \frac{\lambda\kappa P^2(2R^2 r - r^3)}{16\mu^2}, \tag{5}$$

and integrating again equation (5), using boundary conditions $v_c = 0$ at $r = R$, the velocity at the curved part is obtained as

$$v_c = \frac{P}{4\mu}(R^2 - r^2) - \frac{\lambda\kappa P^2}{16\mu^2}\left(\frac{r^4}{4} + \frac{3}{4}R^4 - R^2 r^2\right). \tag{6}$$

This is the velocity in an artery having both curvature and stenosis. This velocity v_c is used again to calculate the volumetric flow rate in the curved region.

C. Volumetric flow rate in a curved artery

Substituting the value of v_c, from equation (6) in the formula for volumetric flow rate Q_c

$$Q_c = \int_0^R 2\pi r v_c \, dr = 2\pi \int_0^R \Bigg(\frac{P}{4\mu}(R^2 - r^2) - \frac{\lambda\kappa P^2}{16\mu^2}\left(\frac{r^4}{4} + \frac{3}{4}R^4 - R^2 r^2\right)\Bigg) r dr,$$

and after integration,

$$\left(\frac{\pi\lambda\kappa}{8\mu^2}\frac{R^6}{6}\right)P^2 - \left(\frac{\pi}{8\mu}R^4\right)P + Q_c = 0. \tag{7}$$

The volumetric flow rate in a curved artery in terms of pressure is

$$Q_c = \frac{\pi P}{8\mu}R^4\left(1 - \frac{P\lambda\kappa}{6\mu}R^2\right). \tag{8}$$

Where there is no curvature i e, $\lambda\kappa = 0$, then Q_c reduces to Q and equation (8) becomes

$$Q = \frac{\pi P}{8\mu}R^4. \tag{9}$$

Note that Q in equation (9) is naturally greater than Q_c in (8). Comparison of these two relations shows that the curvature reduces the volumetric flow rate.

D. Pressure drop and its ratio in the curved part of an artery

Our next attempt is to calculate pressure drop in curved part of the artery having stenosis. We have solved equation (7), which is quadratic in P, and the value is

$$P = \frac{6\mu}{\lambda\kappa\sqrt{\pi}R^3}\left(\frac{\sqrt{\pi}R}{2} \pm \sqrt{\frac{\pi R^2}{4} - \frac{4\lambda\kappa Q_c}{3}}\right). \tag{10}$$

Now onward we take only positive sign while solving the equations and is used to calculate the pressure drop ΔP. For this the pressure P is integrated with respect to z from $-z_0$ to z_0. To reduce the complexity the equation (10) is divided into two parts before integration.

$$P = \frac{3\mu}{\lambda\kappa R^2} + \frac{6\mu}{\lambda\kappa\sqrt{\pi}R^3}\left(\sqrt{\frac{\pi R^2}{4} - \frac{4\lambda\kappa Q_c}{3}}\right). \tag{11}$$

The first term in equation (11) is

$$\frac{3\mu}{\lambda\kappa R^2} = \frac{3\mu}{\lambda\kappa R_0^2}\left(\frac{R}{R_0}\right)^{-2},$$

and in the second term, after binomial expansion of the expression under square root sign up to three terms and is multiplied by the outside term gives

$$\frac{6\mu}{\lambda\kappa\sqrt{\pi}R^3}\left(\frac{\pi R^2}{4}-\frac{4\lambda\kappa Q_c}{3}\right)^{1/2}=\frac{3\mu}{\lambda\kappa R_0^2}\left(\frac{R}{R_0}\right)^{-2} -\frac{8\mu Q_c}{\pi R_0^4}\left(\frac{R}{R_0}\right)^{-4}-\frac{32\mu\lambda\kappa Q_c^2}{3\pi^2R_0^6}\left(\frac{R}{R_0}\right)^{-6}.$$

From equation (2), value of the ratio $\left(\frac{R}{R_0}\right)$ is expanded up to three terms by using binomial expansion and integrated separately and all the four results are added, we get

$$\begin{aligned}\Delta P=&\frac{12\mu z_0}{\lambda\kappa R_0^2}\left(1+2\frac{\delta}{R_0}+3\frac{\delta^2}{R_0^2}\right)\\ &-\frac{16\mu z_0 Q_c}{\pi R_0^4}\left(1+4\frac{\delta}{R_0}+10\frac{\delta^2}{R_0^2}\right)\\ &-\frac{64\mu\lambda\kappa z_0Q_c^2}{3\pi^2R_0^6}\left(1+6\frac{\delta}{R_0}+21\frac{\delta^2}{R_0^2}\right).\end{aligned} \tag{12}$$

After rearranging this equation in the increasing order of δ

$$\begin{aligned}\Delta P=&\frac{4\mu z_0}{R_0^2}\left(\frac{3}{\lambda\kappa}-\frac{4Q_c}{\pi R_0^2}-\frac{16\lambda\kappa Q_c^2}{3\pi^2R_0^4}\right)\\ &+\frac{8\mu z_0\delta}{R_0^3}\left(\frac{3}{\lambda\kappa}-\frac{8Q_c}{\pi R_0^2}-\frac{16\lambda\kappa Q_c^2}{\pi^2R_0^4}\right)\\ &+\frac{4\mu z_0\delta^2}{R_0^4}\left(\frac{9}{\lambda\kappa}-\frac{40Q_c}{\pi R_0^2}-\frac{112\lambda\kappa Q_c^2}{\pi^2R_0^4}\right).\end{aligned} \tag{13}$$

Equation (12) represents pressure drop in curved part of an artery with stenosis. Analysis of this relation is useful to predict the change in pressure drop in the stenosed curved part of an artery. As we know pressure is directly related to the lumen area, which is directly connected to radius of the artery and is effected by the size of the stenosis δ. We can see in this equation each term contains δ and R_0.

If we put $\delta = 0$ in (13), then the equation will represent the pressure where there is no stenosis which is denoted by $(\Delta P)_0$:

$$(\Delta P)_0=\frac{4\mu z_0}{R_0^2}\left(\frac{3}{\lambda\kappa}-\frac{4Q_c}{\pi R_0^2}-\frac{16\lambda\kappa Q_c^2}{3\pi^2R_0^4}\right). \tag{14}$$

Now we can define the ratio $\frac{(\Delta P)}{(\Delta P)_0}$ from equations (13) and (14).

E. Shear stress due to blood flow in a curved artery and its ratio

Shear stress plays highly influencing role in blood flow. In the region of stenosis rough and hard layer helps to increase the shear stress. Using the value of P from equation (10), the shear stress (τ) becomes

$$\tau=\frac{1}{2}Pr=\frac{3\mu r}{\lambda\kappa\sqrt{\pi}R^3}\left(\frac{\sqrt{\pi}R}{2}+\sqrt{\frac{\pi R^2}{4}-\frac{4\lambda\kappa Q_c}{3}}\right). \tag{15}$$

When r is replaced by R_0 we get shear stress at the inner wall of the artery where there is no stenosis. Similarly when r is replaced by R we get shear stress at the inner wall of the stenotic region. We are taking the ratio of the shear stress at non-stenotic to the stenotic region, which becomes

$$\frac{\tau_{R_0}}{\tau_R}=\frac{R^2\left(\frac{\sqrt{\pi}R_0}{2}+\sqrt{\frac{\pi R_0^2}{4}-\frac{4\lambda\kappa Q_c}{3}}\right)}{R_0^2\left(\frac{\sqrt{\pi}R}{2}+\sqrt{\frac{\pi R^2}{4}-\frac{4\lambda\kappa Q_c}{3}}\right)}. \tag{16}$$

When $\lambda\kappa$ is zero and $R = R_0$, then equation (16) gives shear stress ratio in the wall of normal artery, which equals one and can be seen in the description of the shear stress.

III. Results and discussion

Average diameter of left anterior descending and circumflex arteries vary from 3 to 5 mm, so we are taking 4 mm as an average, which makes the radius 2 mm [5]. According to Kashyap et al. [34] when the angle is 80° length of the curvature is 260 mm, for 120°, it is 280 mm. and for 160° it is again 280 mm. Now using the formula $\theta^{\text{rad}} = \frac{\text{arc length}}{\text{radius}}$ the three values of radius of curvature are 0.00537mm^{-1}, 0.00748mm^{-1}, and 0.00998mm^{-1} and their average is 0.00761mm^{-1}.

Actually the artery becomes more curved when the angle between two arms is smaller and the velocity is more affected by the increasing curvature; therefore the curvature works as a deformation factor. From this we conclude that more deformation on artery results, the more decrease in velocity.

According to Klarhofer et al. [37] the range of the velocity 49 mm/sec to 190 mm/sec, so the average time to cover the arc length of 260 mm is approximately 2 sec. Because of this average value for the time variable λ is taken 2 to indicate the time taken by the particle to cover the curvature, which gives the average curvature value $2 \times 0.00761 = 0.015$, and the curvature values are supposed by taking this value as an average value.

In some cases, the curvature values deviate highly from the average value, which are specified for the respective cases. As stenosis increases, the artery radius decreases, so the radius in the stenotic region is the

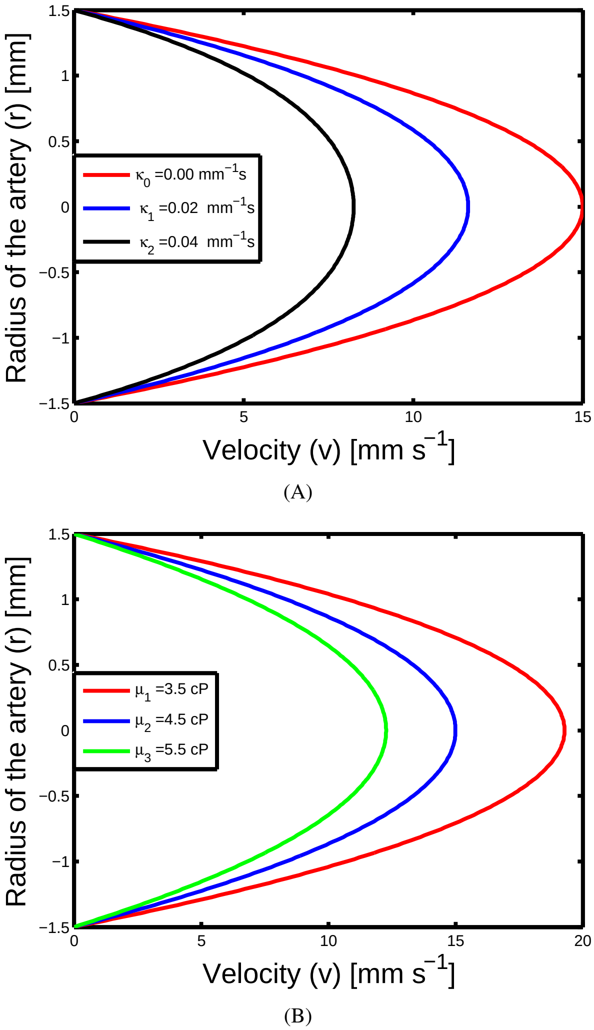


Fig. 2: Variation of velocity with radial distance for different (A) curvature κ and (B) viscosity μ.

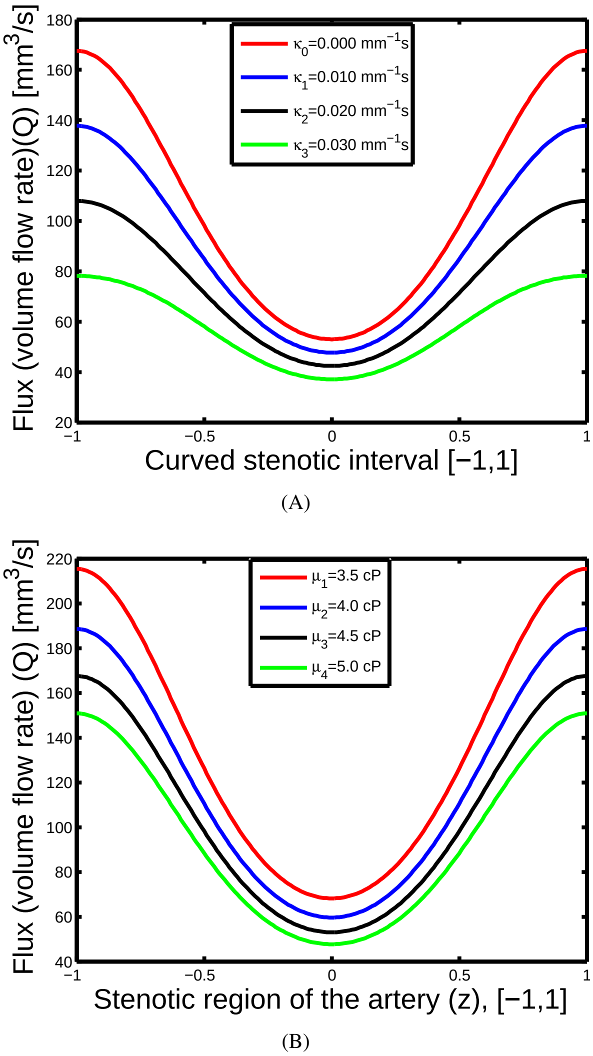


Fig. 3: Relation between volumetric flow rate and increasing height of the stenosis $\frac{\delta}{R_0}$: (A) for different values of curvature κ, (B) for different values of viscosity μ.

difference between the normal artery radius and the height of the stenosis.

In the case of mild stenosis, we have supposed the maximum thickness of the stenosis 0.5 mm and average radius of the artery is in between 1.5 mm. to 2.0 mm. The artery radius is taken 1.5 mm while calculating velocity and shear stress and 2mm for volumetric flow rate and pressure drop, which are mentioned while describing the figures. Again, blood viscosity varies due to shear rate, hematocrit and temperature, but according to Nader et al. [10] its value lies between 3.5 cP to 5.5 cP, so the same range is used in this article.

A. *Velocity profile of blood flow through a curved artery with stenosis*

Figure 2A describes the effect of curvature upon velocity for average viscosity. We have considered the blood pressure 120 mm of Hg, average viscosity 4.5 cP, For this case only the artery radius without stenosis 1.50 mm, maximum height of the stenosis 0.5 mm, which varies according to the values of the cosine function used in the geometry in equation (2). The velocity is measured at the center of the artery along axial direction, where it is maximum and decreases gradually outward from the center. At the beginning for zero curvature the velocity is 15.00 mm s^{-1} which decreases to is 11.62 mm s^{-1}, when the curvature increases to 0.02 mm^{-1}s, and further decreases to 8.245 mm s^{-1} when the curvature rises to 0.04 mm^{-1}s. The velocity decreases from zero curvature to maximum curvature is 6.755 mm s^{-1}, which is 45.033 % of the initial velocity. This shows that the effect of the curvature is significant which reduces the velocity nearly half when the viscosity is 4.5 cP, and the curvature rises from 0 to 0.04 mm^{-1}s. At the point of curvature in an artery, the blood flow experiences large centrifugal forces which push the particles outward from their flowing path, the flow also encounters more resistance and the secondary flow also occurs, which reduces the velocity significantly which is verified by our analytical solution and its graph.

Figure 2B illustrates the effect of viscosity on velocity profile for zero curvature. Blood pressure, artery radius and height of the stenosis are as in the Fig. 2A. To Examine the effect of viscosity only zero curvature is taken and the velocity is measured at the center which, makes easy to compare with the effect of curvature. For the viscosity values 3.5 cP, 4.5 cP, and 5.5 cP, the corresponding velocities are 19.29 mm s^{-1}, 15.00 mm s^{-1}, and 12.27 mm s^{-1} respectively. The decrease in velocity from the minimum viscosity point to the maximum point is 7.02 mm s^{-1}, and in percentage it is 36.391 %. When the result is compared with Fig. 2A, we can see that the effect of curvature is 8.642% higher than the effect of viscosity. This results also suggests that the curvature term incorporated is appropriate to show its effect.

B. Volumetric flow rate of blood flow through curved artery with stenosis

Figure 3A is used to explain the relation between volumetric flow rate and height of stenosis for average viscosity and different values of curvature. Value of the blood pressure, artery size, maximum height of the stenosis are taken as in the case of velocity. To show the curvature effect separately different lines are drawn for different curvature. Maximum height of the stenosis is 0.5 mm and the maximum value of curvature is 0.025 mm^{-1} s. When the curvature is 0.00 mm^{-1}s, the volumetric flow rate at the beginning is 167.60 mm^3 s^{-1} which decreases to 53.01 mm^3 s^{-1} at the peak of the stenosis, which is only 31.62 % of the volumetric flow rate at the beginning, and this is the effect of stenosis only. When the curvature increases to 0.010 mm^{-1}s from 0.00 mm^{-1}s, the volume flow rate decreases from 167.60 mm^3 s^{-1} to 137.80 mm^3 s^{-1}, which is 17.78 % decrease in the original value. The volumetric flow rate at the peak decreases by 29.994% as the curvature increases from 0.00 mm^{-1}s to 0.03 mm^{-1}s. Similarly the volumetric flow rate at the beginning decreases from 167.60 mm^3 s^{-1} to 78.19 mm^3 s^{-1}, which is 53.347 % of the initial volume flow rate. This shows that even if there is no stenosis the volumetric flow rate decreases by 46.653 %, when the curvature increases from0.00 mm^{-1}s to 0.03 mm^{-1}s. The most effective cause for decreasing volumetric flow rate is the decrease in velocity due to curvature by generating the centrifugal force. The centrifugal force creates higher velocity near the outer wall and lower near the inner wall, this uneven distribution of velocity reduces the volumetric flow rate. Secondary flow pattern, increased wall shear stress and flow separation are also responsible to reduce the volumetric flow rate due to curvature which is proved by this graph.

Figure 3B describes the volumetric flow rate for different values of viscosity and zero curvature. Blood viscosity varies within the range, and the other parameters are same as in Figure 3A. Maximum height of the stenosis is 0.5 mm at $z = 0$. All the lines are symmetrically decreasing from -1 to 0 and increasing after 0 because the stenosis is considered symmetric in shape. When the viscosity is 3.5 cP which is minimum, the volumetric flow rate is maximum and it is 215.40 mm^3 s^{-1}, which reduces to 68.16 mm^3 s^{-1} while reaching at the peak of the stenosis. In this case, the volumetric flow rate is reduced by 68.36% compared to its value at the beginning and is equal for every viscosity value within the range. Again we increase the viscosity without increasing the stenotic height, the volumetric flow rate decreases gradually from 215.40 mm^3 s^{-1} to 150.80 mm^3 s^{-1} when the viscosity increases from 3.5 cP to 5.0 cP. The percentage reduction in this case is 29.990% in the initial value. This reduction percentage again verifies that the curvature is more effective than viscosity in case of volumetric flow rate.

Table 1 explains the effect of curvature and stenosis on volume flow rate. In the first row the curvature is zero and the volumetric flow rate is decreased by 68.37 % starting from the point of no stenosis to the peak, this is the effect of stenosis only. In this table curvature values are different from Fig. 3A. In the second row the curvature is 0.010 and at the starting point the volume flow rate is only 137.80 mm ^{3} s^{-1} instead of 167.60 mm ^{3} s^{-1}. This huge decrease in volumetric flow rate is mainly due to curvature only, because all other parameters are same as in the first row. In the second row again when we inspect the

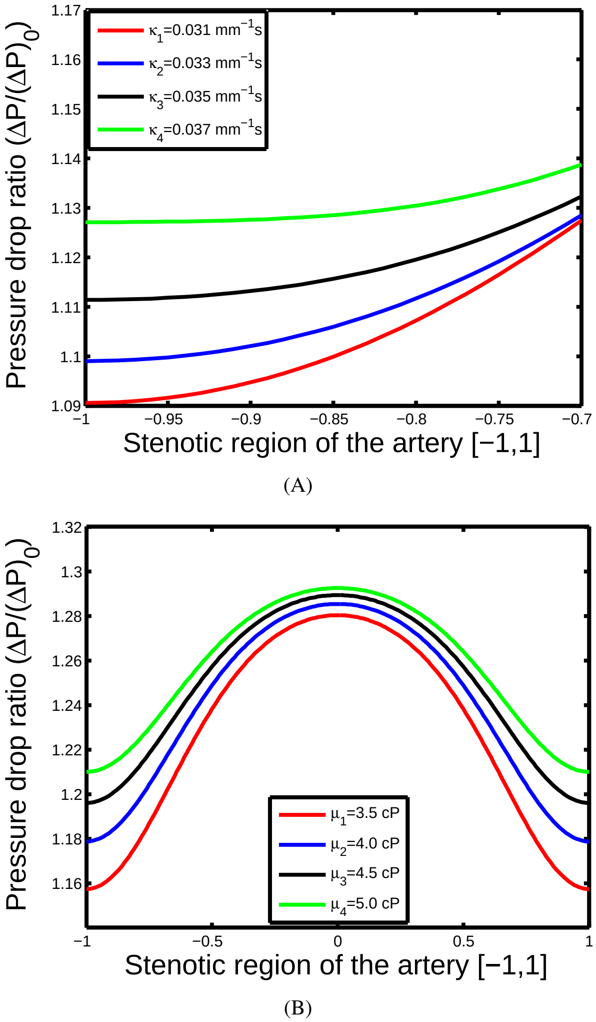


Fig. 4: Relation between ratio of the pressure drop and $\frac{\delta}{R_0}$ for different (A) curvature κ and (B) viscosity μ.

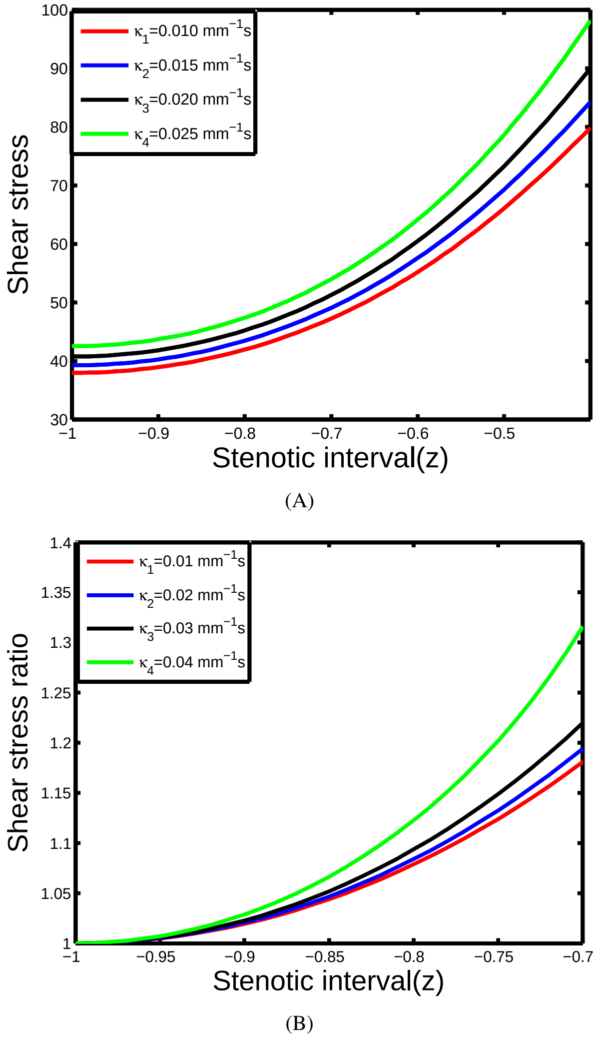


Fig. 5: (A) Shear stress for different curvature along the axial direction. (B) Shear stress ratio for different curvature along the axial direction.

joint effect of stenosis and curvature the volume flow rate decreases from 137.80 mm ^{3} s^{-1} to 47.71 mm ^{3} s^{-1}. When measured in percentage it is 65.37 %, which is also a huge decrease. In the second row, we see that when both stenosis and curvature are present together, the volume flow rate drops by 65.37%, which is quite substantial. Starting from the second row as the curvature is increased by 0.005 mm $^{-1}$ s, the volume flow rate is decreased by 14.9 mm ^{3} s^{-1} in each step. Similarly the decreasing amount at the peak of the stenosis is 2.65 mm ^{3} s^{-1}. Definitely this uniformity in both the points matters. Leaving the first row having zero curvature, the ratio of volume flow rate $\frac{Q_{i+1}}{Q_i}$ at the beginning and at the peak of the stenosis are also almost uniform and not shown in the table. The last two rows show the percentage decrease in volumetric flow rate for increasing curvature and stenosis respectively, which shows that the curvature has a larger effect than the stenosis.

Table 2 describes the effect of viscosity and stenosis upon volumetric flow rate. The blood pressure, radius of the artery and size of the stenosis are same as above. The blood viscosity varies with in the range and average curvature value 0.015 mm^{-1}s is taken to make this table 2. The third column shows the volumetric flow rate decreases gradually with the increasing viscosity, because no stenosis is there. The fourth column shows

Index i, j	Curvature (k_i) [mm^{-1} s]	Volume flow rate at beginning (Q_i) [mm^3 s^{-1}]	Volume flow rate at peak (Q_j) [mm^3 s^{-1}]	$Q_i - Q_j$	$\frac{Q_0-Q_i}{Q_0}$	$\frac{Q_0-Q_j}{Q_0}$
0	0.000	167.60	53.01	114.59	...	...
1	0.010	137.80	47.71	90.09	0.1778	0.09998
2	0.015	122.90	45.06	77.84	0.2667	0.14997
3	0.020	108.00	42.41	65.59	0.3374	0.19996
4	0.025	93.08	39.76	53.32	0.4446	0.24995

Table 1: Analysis of the effect of curvature and stenosis on volumetric flow rate for fixed viscosity.

Index i, j	Viscosity (μ_i) [cP]	Volume flow rate at beginning (Q_i) [mm^3 s^{-1}]	Volume flow rate at peak (Q_j) [mm^3 s^{-1}]	$Q_i - Q_j$	$\frac{Q_0-Q_i}{Q_0}$	$\frac{Q_0-Q_j}{Q_0}$
0	3.5	141.60	55.02	86.58	...	...
1	4.0	131.90	49.58	82.32	0.06850	0.09887
2	4.5	122.90	45.06	77.84	0.13206	0.18103
3	5.0	114.60	41.27	73.33	0.19068	0.24990
4	5.5	107.20	38.05	69.15	0.24294	0.30843

Table 2: Analysis of the effect of viscosity and stenosis on volumetric flow rate for fixed curvature.

the effect of both viscosity and stenosis upon volumetric flow rate, at the peak of the stenosis. Fifth column is the difference in volumetric flow rate at the beginning and at the peak of the stenosis. In the second last column (or the sixth column), the volumetric flow rate is compared with the initial value where there is no stenosis and the last column is the comparison with the initial value at the peak of the stenosis. Comparing the results of the last two columns, the percentage decrease in volumetric flow rate is greater when the effect of both viscosity and stenosis is considered. We have also calculated the ratio $\frac{Q_{i+1}}{Q_i}$, at the beginning and at the peak of the stenosis, and it is approximately 0.93. The percentage decrease in volume flow rate is about 7 % for 0.5 cP increase in viscosity within the range we have considered.

C. Pressure drop of blood flow through a curved artery with stenosis

Figure 4A explains the change in pressure drop ratio in the stenotic region for different values of curvature. In this case the blood pressure is 120 mm of Hg, R is 2 mm. The volumetric flow rate Q varies with the height of the stenosis, the radius R along the axial direction decreases gradually from -1 to 0, which reduces Q and it affects the pressure drop. The pressure drop ratio is measured at the beginning and at the peak of the stenosis. When the curvature value is 0.030 mm^{-1}s, pressure drop ratio at the beginning is 1.091, which increases to 1.127 at the peak. Similarly it is 1.099, 1.111, and 1.127 at the beginning for the curvature values 0.032 mm^{-1}s, 0.034 mm^{-1}s and 0.036 mm^{-1}s respectively. For the same values of curvature the pressure drop ratio at the peak of the stenosis are 1.128, 1.132, and 1.139 respectively. The pressure drop ratios are increasing continuously from beginning to the peak of the stenosis. The increment in pressure drop ratio are 3.299 %, 2.638 %, 1.890 % and 1.064 % for the curvature values 0.030 mm^{-1}s, 0.032 mm^{-1}s, 0.034 mm^{-1}s, and 0.036 mm^{-1}s respectively. When the curvature increases from 0.030 mm^{-1}s, to 0.036 mm^{-1}s the pressure drop ratio at the beginning increases by 3.299% and 1.064 % at the peak. Finally we have calculated the combined effect of the curvature and stenotic height, the pressure drop ratio for minimum curvature and zero stenosis is 1.091 which increases to 1.139 for maximum curvature and at the peak of the stenosis, which is 4.399% increment in the initial value. The results indicate a significant effect of curvature; however, they are valid only for a specific range of curvature values and not for all values of curvature. Das et al. [19] have considered fixed curvature, and according to Kashyap et al. [34] the pressure gradient increases with the curvature.

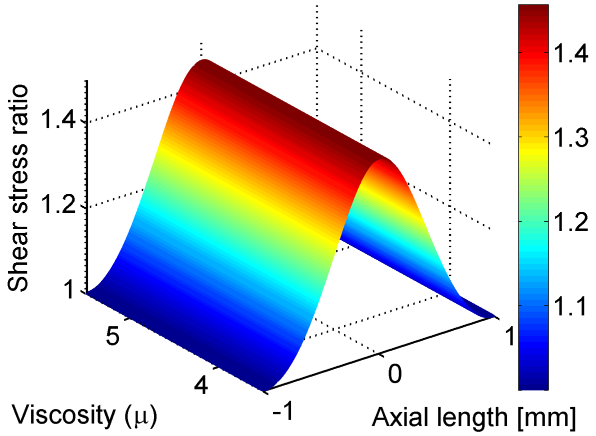
Figure 4B describes the relation between pressure drop ratio and viscosity for fixed curvature 0.0125, which is the average value of the curvature we have considered. In this case the pressure is 120 mm of Hg, R is 2 mm, maximum thickness of the stenosis is 0.5 mm. The pressure drop ratios are measured at the beginning and at the peak of the stenosis. For the minimum viscosity of 3.5 cP, the pressure drop ratio at the beginning is 1.157, which increases to 1.280 at the peak of the stenosis, which is 10.630% increment in the previous value. Similarly for the maximum viscosity of 5.0 cP, the pressure drop ratio at the beginning is 1.210, which increases to 1.293 at the peak of the stenosis, and this increment is 6.859% of the initial value. The percentage increment is more when there is minimum viscosity, and increment percentage decreases from 10.63 % to 6.859 % when the viscosity increase from 3.5 cP to 5.0 cP. From this it is concluded that the pressure drop increases for increasing viscosity and increasing height of the stenosis but the percentage of increment decreases gradually for higher viscosity.

D. Shear stress ratio of blood flow in a curved artery with stenosis

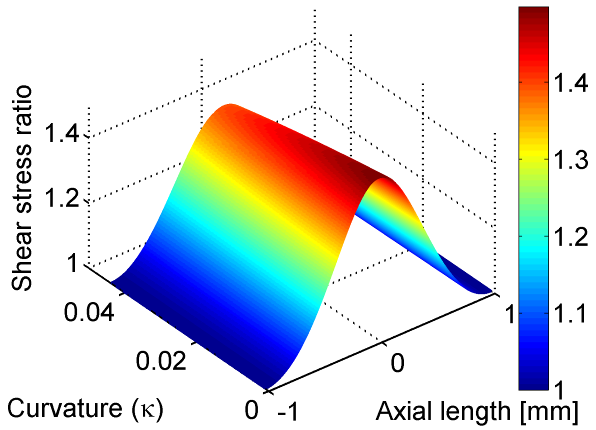
Figure 5A describes the relation between stenotic height and shear stress for different curvature. This Fig. shows that the shear stress increases with stenotic height and also with the increasing curvature. Shear stress values are measured at the beginning and at a distance of 0.4 mm after starting the stenosis. A huge increment is seen as we move towards the peak of the stenosis. When the curvature is 0.010 mm^{-1}s the wall shear stress at the beginning is 37.94 cP which increases to 79.78 cP when it covers a distance of 0.4 mm. This increment is 110.27%, and for the curvature 0.025 mm^{-1}s the percentage increment becomes to 130.894%. The increment percentage at the beginning when there is no stenosis is 12.018%, and when both stenosis and curvature are included with fixed stenosis of height 0.4 mm, the percentage increment becomes 23.00%. This result shows that the joint effect of curvature and stenosis is more dangerous, and the flow is disturbed near the peak of the stenosis.

Figure 5B describes the relation between stenotic height and shear stress ratio for different curvature. At the beginning both radii with and without stenosis are equal so the ratio starts from one. But the radius in the stenotic region changes and the ratio increases gradually. When the curvature value is 0.010 mm^{-1}s the ratio increases from 1 to 1.180 which is 18.00% increment in the initial value. When the curvature is increased from 0.010 mm^{-1}s to 0.020 mm^{-1}s the shear stress ratio increases to 1.194 which is the increment of 1.186% in the initial value. Similarly the increasing percentage are 3.305% and 11.440% for the curvature 0.030 mm^{-1}s and 0.040 mm^{-1}s respectively. In this case, the flow becomes disturbed near the peak of the stenosis, and usual rules break near and at the peak.

Fig. 6A describes the shear stress ratio due to stenotic height and viscosity near the inner wall of the artery. Shear stress ratio increases gradually along the axial direction with stenotic height, and then decreases after the peak. When viscosity is 3.5 cP, at the peak the shear stress ratio is 1.456 which increases to 1.457 for the viscosity 5.5 cP which shows a small increment due to increase in viscosity.



(A)



(B)

Fig. 6: (A) Shear stress ratio due to stenotic height and viscosity. (B) Shear stress ratio due to stenotic height and curvature.

Fig. 6B shows the shear stress ratio due to stenotic height and curvature. Starting from one at the bottom, the shear stress ratio increases gradually with stenosis and reaches 1.493 at the peak, when the curvature is zero. Then after we increase curvature from 0.00 mm^{-1}s to 0.04 mm^{-1}s, and the shear stress ratio at the peak increases to 1.393, this shows that the ratio increases due to curvature is less by 0.1 than the viscosity.

IV. Summary

Curvature in an artery is very common and significantly affects flow dynamics. The curved part of an artery is the most favorable for atherosclerosis, so the present work is focused on the curved stenotic artery. A quadratic curvature term is incorporated in the Navier-Stokes equation to develop the new model based on the principle of centrifugal force.

Flow parameters, such as velocity profile, volumetric flow rate, pressure drop ratio, shear stress, and shear stress ratio of the new model, are analytically derived and analyzed in detail. When viscosity increases from minimum to its maximum value, the velocity is reduced by one third nearly, but the curvature reduces nearly half, so the curvature is more significant than viscosity.

The volumetric flow rate in the axial direction decreases with increasing curvature and viscosity, a small amount of curvature also makes a significant impact in volumetric flow rate. In this case as well, the effect of curvature is more pronounced than viscosity. The decreasing rate of volumetric flow rate is almost uniform due to the effect of curvature as well as viscosity. The pressure drop ratio increases with increasing curvature along the axial direction and the complete disturbance occurs near the peak of the stenosis.

Shear stress ratio increases with increasing stenosis and the increment becomes exponential near the peak. This result indicates that the effect of curvature is more significant when compared with the viscosity. In the geometrical interpretation the interval from the beginning to the end point is symmetric in shape which is due to the cosine function used to represent the stenosis. The portion having both stenosis and curvature is very sensitive and affects significantly the blood flow system. A small change in curvature has brought subsequent changes in hemodynamics which indicates the sensitiveness of the curvature term added in it.

This model can be a useful numerical tool to know the accuracy of blood flow through a stenosed, curved artery. It will help researchers in this sector and it can be used clinically to investigate the disorders in the blood flow and to identify the defects in the cardiovascular system.

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