The effect of conspecific support on the asymptotic properties of competitive systems

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In competitive system the long term dynamics, e.g. co-existence, extinction, are usually determined by the demographics of the competing species as well as the effect of their interaction on each one. Here we consider the special case when the competing species have the same demographics and the interaction has the same impact on each one. Assuming an underlying logistic model

$$\dot{x} = x(1-x) \tag{1}$$

for each population and a mass action type of interaction, the model of two competing species can be represented in the following non-dimensional form

$$\dot{x} = x(1-x-y), \quad \dot{y} = y(1-x-y) \tag{2}$$

In this form the model can be interpreted as model of two species competing for a common resource. The model (2) has infinitely many stable equilibria of co-existence, $\{(x,y) : x+y=1,\ x>0,\ y>0\}$. In fact, from modelling point of view, the two species are indistinguishable and can be considered as one species. Indeed, $x+y$ satisfies (1). The situation changes dramatically if conspecific support is introduced, that is individuals of one species support their own but not the other species. The benefit of conspecific support is one of the essential mechanisms for the Allee demographic effect - positive correlation between the per capita growth rate and the population size. The conspecific support can be modeled in different ways. Here we use a power function. Assuming that the density independent death rate is μ, a conspecific support changes model (1) to

$$\dot{x} = x^{1+\alpha}(1+\mu-x) - \mu x \tag{3}$$

Clearly, (1) is a particular case of (3) when $\alpha = 0$. Further, (3) has the same positive stable equilibrium as equation (1), namely $x = 1$. However, away from this equilibrium the dynamics of (3) are quite different from the dynamics of (1). Specifically, the extinction equilibrium is stable and there is a positive unstable equilibrium, often referred to as a minimum survival level. With conspecific support, the system (2)changes to

$$\dot{x} = x^{1+\alpha}(1+\mu-x-y) - \mu x, \quad \dot{y} = y^{1+\alpha}(1+\mu-x-y) - \mu y \tag{4}$$

We show in this presentation that if $\alpha > 0$ for any non-equilibrium initial state of the system at least one of the species goes extinct. More precisely, we prove that system does not have any strictly positive stable equilibria. The stable equilibria are $(0,0)$, $(1,0)$ and $(0,1)$. The closures of their basins of attraction completely cover $\mathbb{R}^2_+$.