Optimal control of an age-structured SVIRS model with treatment and vaccination

Xi-Chao Duan[1], Xue-Zhi Li[2], Maia Martcheva[3]
[1] School of Mathematical Sciences, Tongji University, Shanghai 200092, China
xcduan82@126.com
[2] College of Mathematics and Information, Henan Normal University, Xinxiang 453007, China
xzli66@126.com
[3]University of Florida, Gainesville, FL 32611, USA
maia@ufl.edu

Keywords: epidemic model, age-since-recovery, age-since-vaccination, vaccination, optimal control

We develop and analyze an age-structured SVIRS epidemic model with ages of vaccination [1] and recovery which allows for variations in the exit rate of the removed as a function of age (time since recovery) and variations in the vaccine loss rate as a function of age (time since vaccination). We show that the age-structured model has one disease-free steady state and an endemic steady state. Local and global stability analysis show that the disease-free steady state is locally and globally stable if the reproduction number is below one. This means that if the reproduction number is reduced below one, say through vaccination or treatment, the disease will be eliminated. In addition, to provide a better understanding of the interaction between treatment and vaccination, we formulate an optimal control problem [2] for the age-structured model and then derive the existence of solutions to the optimal control problem from optimal control theory. The optimal treatment and vaccination strategies are obtained by solving the corresponding optimality system numerically. It is demonstrated by numerical simulations that the effectiveness of the optimal therapy and vaccination protocols would be strongly affected by the recovery age of the removed.

References

[1] Iannelli M., Martcheva M., Li X.-Z., *Strain replacement in an epidemic model with super-infection and perfect vaccination*, Mathematical Biosciences, *195 (1) (2005)*, 23–46.

[2] Blayneh K., Cao Y., Kwon H-D., *Optimal control of vector-borne diseases: treatment and prevention*, Discrete Continuous Dyn Syst B, *11(3) (2009)*, 587-611.