Homoclinic and torus bifurcations on cells repair regulations model of the metastatic nasopharyngeal carcinoma

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Keywords: NPC, metastasis, bifurcation, torus, homoclinic orbit

We consider a mathematical model of the Nasopharyngeal Carcinoma (NPC) that grows on the nasopharynx epithelial cell. The model shows the interaction between ATM, p53, MDM2, and DSB as the proteins that play important roles in the cells repair regulations for the metastatic NPC. Our study is motivated by the result in [1] where there are some irregular behavior appear through the mutation of the proteins in the metastases of NPC. In this paper we will study the appearance and the mechanisms of chaos for the metastatic NPC case. We employ the bifurcation analysis to determine the mechanism of chaos for our system and we find two important bifurcations, i.e., the Zero Hopf and Generalized Hopf bifurcations. These two bifurcations occur simultaneously when the ATM protein, which binds Wip1 protein with high affinity, is degrade and p53 ubiquitination by MDM2 has increasing rate. The Generalized Hopf bifurcation also occurs in two other cases. First, it occurs when the induction rate of DSB by EBNA1 and the maximal rate of the ATM dephosphorylation by Wip1 increase. In the second case, the bifurcation occurs when the ATM and p53 proteins that bind Wip1 protein with high affinity are degraded, and the MDM2 protein that bind the phosphorylated Akt with high affinity has increasing rate. The homoclinic orbit and torus as an indication of chaos are triggered those bifurcations. The interpretation of the results is based on the information in [2].

References

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