Dynamical behaviour of tuberculosis transmission

Dhiraj Kumar Das[1], Subhas Khajanchi[2], T. K. Kar[3]
[1] Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah 711103, India
dhiraj.hmc@gmail.com
[2] Department of Mathematics, Bankura University, Bankura - 722155, India
subhaskhajanchi@gmail.com
[3]Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah 711103, India
tkar1117@gmail.com

Keywords: Tuberculosis model, Basic reproduction number, Backward bifurcation, Hopf-bifurcation.

Tuberculosis(TB) is a contagious disease in human caused by infection with Mycobacterium tuberculosis(*Mtb*). Most infections results a clinically asymptotic state termed as latent TB infection(LTBI) whereas a smaller portion of infected individuals grow symptomatic active pulmonary TB. The main difference between TB and other infectious diseases is that, the disease progression from primary infection(LTBI) to active pulmonary TB is significantly time-consuming.

We proposed and study an SEIR type mathematical model for TB transmission incorporating roles of both exogenous re-infection and endogenous re-activation. Our model possesses two kinds of steady states: infection free and endemic. The epidemiological threshold key *that is*, basic reproduction number R_0 has been obtained by using next-generation matrix. We observe that the disease transmission rate β and exogenous re-infection level α plays a significant role in order to determine the qualitative behaviour of our proposed model system. Our results demonstrate that when exogenous re-infection level crosses a critical value our system undergoes backward bifurcation and hence a stable endemic equilibrium exits in spite of the fact $R_0 < 1$. Therefore, reducing R_0 less than unity is not sufficient to eradicate TB completely. We further investigate that proposed model experience stable periodic solutions as β increases through a critical value. Various numerical simulations have been conducted covering the breadth of feasible parameter space to support analytical establishments.

References

[1] S. Khajanchi, D.K. Das, T.K. Kar *Dynamics of tuberculosis transmission with exogenous reinfections and endogenous reactivation*, Physica A (2018) **https://doi.org/10.1016/j.physa.2018.01.014**