Comparison of some numerical methods for the FitzHugh-Nagumo equation

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The FitzHugh-Nagumo equation has various applications in the fields of flame propagation, logistic population growth, neurophysiology, autocatalytic chemical reaction and nuclear theory [1,2]. It is given by $u_t - u_{xx} = u(1-u)(u-\gamma)$, where $\gamma \in (0,1)$ and $u(x,t)$ is the unknown function which depends on spatial variable x and temporal variable t. Namjoo and Zibaei [3] solved the equation above using a new scheme which they baptised as 'exact' scheme and the following initial boundary conditions are used:

$$u(x,0) = \frac{\gamma}{1+e^{-2A_1 x}},\ 0 \leq x \leq 1,\ A_1 = \frac{\sqrt{2}}{4}\gamma,$$
$$u(0,t) = \frac{\gamma}{1+e^{2A_1 A_2 x}}\ u(1,t) = \frac{\gamma}{1+e^{-2A_1(1-A_2 t)}},\ t \geq 0,\ A_2 = \frac{4-2\gamma}{\gamma}A_1.$$

The values they used are in [3] as $\gamma = 0.001$, temporal step size $\Delta t = 0.001$, spatial mesh $\Delta x = 0.001$ and they obtained quite accurate results. In their derivation of the 'exact' scheme, the following condition is used namely $\Delta x = A_2 \Delta t$. If we choose $\gamma = 0.001$, $\Delta x = 0.1$, this would mean that $\Delta t = 0.07$. However if we choose $\gamma = 0.001$, $\Delta x = 0.1$, the scheme works with some values of Δt such as 0.0005, 0.001, 0.002, 0.004, 0.008 but the scheme is not stable for $\Delta t > 0.008$. We first attempt to explain why the scheme derived is problematic. We then derive some non-standard finite difference scheme and compare the results with those from a classical scheme and 'exact' scheme of Namjoo and Zibaei. We work with some different values of γ such as 0.001, 0.5. We compare the stability region and the rate of convergence of the methods.

[1] Wazwaz, Abdul-Majid and Gorguis, Alice, *An analytic study of Fisher's equation by using Adomian decomposition method*, Applied Mathematics and Computation, 609-620, 2004.

[2] Bhrawy, AH, *A Jacobi–Gauss–Lobatto collocation method for solving generalized Fitzhugh–Nagumo equation with time-dependent coefficients*, Applied Mathematics and Computation, 255–264, 2013.

[3] Namjoo, Mehran and Zibaei, Sadegh,*Numerical solutions of FitzHugh–Nagumo equation by exact finite-difference and NSFD schemes*, Computational and Applied Mathematics, 1-17, 2016.