Analysis of chemostat models with application to a model of anaerobic digestion

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Keywords: chemostat, continuous stir tank reactor, non-monotone response function, species decay rate, stochastic simulations, biogas production, anaerobic digestion, global stability analysis.

The talk will begin by providing a short, apparently overlooked, proof of competitive exclusion for a model of n-species competition in the chemostat when maintenance is ignored [3]. Then a simplified model of anaerobic digestion will be considered. Anaerobic digestion is a complex naturally occurring process used for waste and wastewater treatment to produce biogas as a renewable source of energy. The so-called Anaerobic Digestion Model No. 1 (ADM1) in [1] includes 32 state variables and is not mathematically tractable. Bornh oft et al. [2] proposed a simplified model that seems to capture most of the qualitative dynamics of the ADM1 model, including the possibility of bi-stability and the bifurcation dynamics when substrate concentration is used as the bifurcation parameter. Our analysis shows that not all of the dynamics possible in ADM1 are captured. Our analysis of this model also required studying a chemostat model without ignoring the maintenance term for a general class of response functions, that includes nonmonotone functions, that had not been analyzed. I will provide a proof in this case for competition between two populations using the Lyapunov function introduced in [4]. The talk will conclude with a consideration of the sensitivity of the predictions of the model to stochastic perturbations,and a discussion of the implications of the model predictions for successful operation of the process of anaerobic digestion.

References

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