A reconstruction of the first FSSP algorithm

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The firing squad synchronization problem (FSSP) on cellular automata has been studied extensively for more than fifty years, and a rich variety of synchronization algorithms has been proposed. Goto's FSSP algorithm (Goto [1962]) has been known as the first minimum-time FSSP algorithm, however the paper itself had been a completely unknown one in the research community of cellular automata for a long time due to its hard accessibility. In the present paper, we reconstruct the Goto's FSSP algorithm and present the first small-state implementation. The implementation is realized on a cellular automaton having 165-state and 4378 transition rules and the realization is far smaller than Goto [1962] imagined, where he thought that it would require many thousands of thousands states. It is shown that the reconstructed algorithm uses a quite different synchronization mechanism in comparison with the designs employed in Waksman [1966], Balzer [1967], Gerken [1987], and Mazoyer [1987]. We show that the algorithm has $\Theta(n \log n)$ minimum-state-change complexity for synchronizing n cells. The algorithm is optimum not only in time but also in state-change complexities. We show that the reconstructed algorithm can be generalized as to the initial general's position and its implementation on a cellular automaton with 434 internal states and 13328 state-transition rules is also given. The general purpose of this investigation is to achieve more insights into the structure of the classical minimum-time FSSP solutions and such insights would be helpful in the design of new FSSP algorithms.

References

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