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Higher-order modified nonstandard finite difference methods for autonomous dynamical systems

Fawaz K. Alalhareth[1], Madhu Gupta[2], Hristo V. Kojouharov[3], Souvik Roy[3]

[1]Department of Mathematics, College of Science and Arts,
Najran University, Najran, Saudi Arabia
fkalalhareth@nu.edu.sa

[2]Mathematical Sciences Department,
George Mason University, Virginia, USA
mgupta25@gmu.edu

[3]Department of Mathematics,
The University of Texas at Arlington, Arlington, Texas, USA
hristo@uta.edu
souvik.roy@uta.edu

Mathematical models in biology and medicine help to better understand and accurately predict the dynamic processes involved in complex biological systems at the molecular, cellular, and organism levels. One powerful tool for modeling these systems is the use of autonomous dynamical systems. For example, in ecology, autonomous dynamical systems arise when modeling interspecies interactions, and they describe the rates of change in the population size of each interacting component. Designed to approximate specific dynamical systems, nonstandard finite-difference (NSFD) methods preserve some of the essential structural properties of the approximated system for arbitrary step-size, avoid some of the major numerical restrictions, and improve the computational efficiency. The nonstandard discretization rules were pioneered by R. E. Mickens [1] and usually include non-local discrete representations of nonlinear terms in the right-hand side of the systems and variations in the representations of the time derivatives. Series of papers have been dedicated to the development of elementary stable nonstandard (ESN) methods and positive and elementary stable nonstandard (PESN) methods, which preserve two of the most important qualitative properties of biological models, namely unconditional positivity of all feasible trajectories and local stability of existing equilibrium points. However, they are generally only of first-order accuracy.

In this project, we extend our previous work on ESN and PESN methods to develop four new classes of modified NSFD methods that are not only elemen-

tary stable and preserve the positivity of solutions, but also have second-order accuracy [2]. The proposed modified NSFD methods use a novel modified nonstandard denominator function in the discretization of the derivative, that is dependent not only on the step-size but also on the numerical solution.

We also present a set of numerical simulations for select problems in biology that support the theoretical results and demonstrate the superior performance of the proposed new methods over other classical standard and nonstandard numerical methods.

Keywords: nonstandard, finite difference, positivity, elementary stable, second-order, NSFD, ESN, PESN

MSC2020: 65L05, 65L20, 65L06, 65L12

References

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