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Mathematical modeling of trace-metals precipitation in biofilms

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Biofilms are colonies of microorganisms embedded in a matrix of extracellular polymeric substances (EPS). They play major roles in many fields such as biotechnology and health [1].

Mathematical modelling is an essential tool in understanding biofilms and their interactions with the media in which they evolve and in particular with inorganic materials because it reduces experimental testing and scale up [2].

In this work, we present a mathematical model that describes the growth of a biofilm and the precipitation of trace-metals within the biofilm. In contrast to existing works in the literature (e.g. [3, 4, 5]), our model takes into consideration the occupation of the liquid phase (porosity) of the biofilm by the precipitates that form during the growth of the biofilm.

More specifically, the general formulation of this model includes:

- A system of first order quasi-linear hyperbolic equations that model the biomass growth, the accumulation of precipitates, and the porosity. The source terms of the porosity and precipitation equations are formulated so that the space occupied by the precipitates and porosity remains constant over time.
- A system of second order parabolic equations modeling the diffusion of biomass nutrients, cations, and anions which combine with cations to form precipitates. We introduced a novelty by considering the fact that anions can be produced both by diffusion and by the metabolism of bacteria.

- A nonlinear ordinary differential equation which is the free boundary of the problem and takes into account the temporal evolution of the biofilm thickness.

The entire model is therefore a free boundary problem that can be adapted to any type of biofilm (including granules) and trace-metals. A numerical application of the model is proposed. Apart from the precipitation in metals in the biofilms, it includes the competition existing between the sulfate-reducing bacteria and the methanogens which are bacteria involved in the last phase of the methane production (methanogenesis). Our numerical simulations show that the concentration of precipitates is not uniformly distributed as suggested by experimental studies in [6]. Indeed, the model shows that precipitation occurs more at the bottom of the biofilm than it does at the surface in agreement with experiments.

Keywords: Biofilms, Precipitation, Hyperbolic PDE, Parabolic PDE

References

[1] H.-C. Flemming, J. Wingender, U. Szewzyk, P. Steinberg, S. A. Rice, S. Kjelleberg, Biofilms: an emergent form of bacterial life, *Nature Reviews Microbiology*, 14:563-575, 2016.

[2] M. A. Delavar, J. Wang, *Advanced Methods and Mathematical Modeling of Biofilms: Applications in health care, medicine, food, aquaculture, environment, and industry*, Elsevier, 2022.

[3] C. K. Mbamba, S. Tait, X. Flores-Alsina, D. J. Batstone, A systematic study of multiple minerals precipitation modelling in wastewater treatment, *Water Research*, 85:359-370, 2015.

[4] T. Zhang, I. Klapper, Mathematical model of biofilm induced calcite precipitation, *Water Science and Technology*, 61(11):2957-2964, 2010.

[5] T. Zhang, I. Klapper, Mathematical model of the effect of electrodiffusion on biomineralization, *International Journal of Non-Linear Mechanics*, 46(4):657-666, 2011.

[6] Y. Bai, X.-J. Guo, Y.-Z. Li, T. Huang, Experimental and visual research on the microbial induced carbonate precipitation by *Pseudomonas aeruginosa*, *AMB Express*, 7:57, 2017.