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# On reliable numerical methods for some real-life differential equation models with singularities

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The Finite Difference Method (FDM) and the Finite Element Method (FEM) are among the most used techniques for numerical solutions of ordinary and partial differential equations. Of paramount importance is the construction of FDM and FEM that are reliable in that they permit to gain useful insight on the solutions of the continuous differential equations being studied. For boundary value problems for partial differential equations with domain-singularities such as corners, vertices and edges, there has been considerable efforts to construct innovative FEMs in which the optimal rate of convergence, with error estimates, is restored. To the convergence of a numerical method, the Nonstandard Finite Difference (NSFD) method created by R Mickens in the late eighties brings a further level of reliability of a numerical scheme, namely that it should be dynamically consistent with respect to the involved continuous differential equation [1]. The FEM or its variants and the NSFD method are abundantly used in Biosciences, the focus of this international conference, see for instance [2, 3].

The purpose of this presentation is twofold.

1. We give an overview of our work, [4, 5, 6], on reliable FEM and NSFD schemes in three settings of differential equations with different types of singularities, which are relevant in Mathematical Biology. The first setting is that of a one-dimensional singularly perturbed boundary value problem. We design a Singular Function Method (SFM) and a Mesh Refinement Method (MRF), which are proved to be theoretically and computationally uniformly convergent with respect to the perturbation parameter. The one-dimensional Burgers equation constitutes the second setting. We construct a NSFD scheme that preserves the boundedness and monotone decreasing properties of the kinetic energy. Finally, in the setting of a linear reaction-diffusion equation on a nonsmooth domain, we establish a bridge between the two reliable approaches by coupling NSFD discretizations in the time variable with the SFM or MRM in the space variables.

2. Whenever the data functions of the differential equations are smoother, we establish and restore optimal higher rates of convergence of the schemes despite the presence of singularities.

*Keywords: Finite element method, Nonstandard finite difference schemes, Singularities*
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