



## Dynamical analysis combined with parameters identification for a model of infection in honeybee colonies with social immunity

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Several models on honeybee population dynamics have been considered in the past decades, which explains that the growth of bee colonies is highly dependent on the availability of food and social inhibition. The phenomenon of the Colony Collapse Disorder (CCD) and its exact causes remain unclear and here we are interested on the factor social immunity.

We work with the mathematical model in [1]. The core model, consisting of four nonlinear ordinary differential equations with unknown functions: brood and nurses  $B$ ,  $iB$ ,  $N$  and  $iN$  represent the number of healthy brood, infected brood, healthy nurses, and infected nurses, respectively.

First, this model implements social segregation. High-risk individuals such as foragers are limited to contact only nectar-receivers, but not other vulnerable individuals (nurses and brood) inside the nest. Secondly, it includes the hygienic behavior, by which healthy nurses actively remove infected workers and brood from the colony.

We aim to study the dynamics and its long-term behavior of the proposed model, as well as to discuss the effects of crucial parameters associated with the model. In the first stage, we study the local stability of the model around each equilibrium points in dependence of the reproduction number.

In the second stage, we investigate the inverse problem of parameters identification in the model based on finite number time measurements of the population size. The conjugate gradient method with explicit Frechet derivative of the cost functional is proposed for the numerical solution of the inverse problem.

Computational results with synthetic and realistic data are performed and discussed.

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## References

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