



## Pursuit-evasion dynamics in predator-prey models

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We study a pursuit-evasion predator-prey model

$$\begin{cases} P_t = d_P \Delta P - \xi \nabla \cdot P \nabla N + f(P, N), \\ N_t = d_N \Delta N + \chi \nabla \cdot N \nabla W + g(N, P), \\ W_t = d_W \Delta W + \gamma P - \mu W, \end{cases} \quad (1)$$

where  $P$  and  $N$  denote predator and prey densities while  $W$  is the chemical density. The reaction functions  $f$  and  $g$  describe local predator-prey interactions and birth/death processes,  $d_P, d_N, d_W > 0$  are diffusion constants,  $\xi, \eta > 0$  are taxis sensitivity parameters,  $\gamma$  and  $\mu$  are rate coefficients related to the production and degradation of the chemical secreted by the predators. The taxis term in the first equation describes direct prey taxis, i.e. the movement of predators towards the density gradient of prey (pursuit) while the second equation represents situation in which the prey senses not the presence of predators themselves but rather their odor, a diffusive chemical with density  $W$  secreted by the predators so that the prey use evasive strategy moving in the opposite direction with respect to the gradient of  $W$ . System (1) is supplemented by initial conditions and no-flux boundary conditions describing the lack of migration through the boundary of a region where the species under consideration are distributed.

We shall present results published in [1, 2] and some yet unpublished on the existence of global-in-time solutions and formation of space-time patterns for the range of parameters when a space-homogeneous coexistence steady state loses its stability. On assuming the reaction part of the model of the classical Bazykin type we find some biologically relevant modifications in the taxis part of the model such that the blow-up of solution in finite time is prevented for a broad class of initial data.

The results are illustrated by various numerical simulations.

**References**

- [1] P. Mishra, D. Wrzosek, Pursuit-evasion dynamics for Bazykin-type predator-prey model with indirect predator taxis, *Journal of Differential Equations*, 361:391-416, 2023.
- [2] P. Mishra, D. Wrzosek, Repulsive chemotaxis and predator evasion in predator-prey models with diffusion and prey-taxis, *Mathematical Models and Methods in Applied Sciences*, 32(1):1-42, 2022.