**Mathematical Methods and Models in Biosciences**
June 18-23, 2023, Pomorie, Bulgaria
https://biomath.math.bas.bg/biomath/index.php/bmcs


# A stochastic model of seasonal savanna

Paweł Klimasara[1], Marta Tyran-Kamińska[2]

[1]University of Information Technology
and Management in Rzeszów, Poland
pklimasara@wsiz.edu.pl

[2]Institute of Mathematics,
University of Silesia in Katowice, Poland
mtyran@us.edu.pl

Savannas are mixed forest-grassland ecosystems that cover around 20% of the Earth's surface. They occur in areas with a distinct dry season and a rainy season. In understanding the complex dynamics of savannas, a major concern is how do trees and grasses coexist without one dominating the other? Random fires are believed to be key. From a mathematical point of view, most savanna modeling attempts in the literature do not consider a stochastic approach to fire occurrence, and the occurrence of seasons in deterministic models is often analyzed only numerically [1].

We propose a stochastic savanna model [2], which is formally a hybrid process described by piecewise-deterministic Markov processes [3] reflecting repeated switching between two seasons. Such a description requires an additional time variable to track the length of stay in the current season, leading to time-homogeneous Markov processes. We examine the time averages of their distributions and give sufficient conditions for their convergence. Although we focus on the savanna dynamics model as an example, we present a general theory that can be used for other formally similar models or in situations where there are more than two seasons.

**References**

[1] E. R. White, A. Hastings, Seasonality in ecology: Progress and prospects in theory, *Ecological Complexity*, 44:100867, 2020.

[2] P. Klimasara, M. Tyran-Kamińska, A model of seasonal savanna dynamics, *SIAM Journal on Applied Mathematics*, 83(1):122-143, 2023.

[3] R. Rudnicki, M. Tyran-Kamińska, *Piecewise Deterministic Processes in Biological Models*, Springer, Cham, 2017.