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Mathematical and numerical methods for understanding immune cell motion during wound healing

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We propose a new workflow to analyze macrophage motion during wound healing. These immune cells play a critical role in tissue repair, as they are attracted to the wound site after an injury. Their motion is a combination of directional movement and random motion. Therefore, we begin by smoothing the original trajectories. The smoothing model is based on curve evolution approach, where the curve evolves under the influence of two key terms: a smoothing term, determined by the local curvature of the trajectory, and an attracting term, which ensures that the curve stays close to the original trajectory.

This model allows us to separate the random parts of motion from the directional parts and study them separately. Once the random sub-trajectories are obtained, their properties are analyzed using the mean squared displacement. This method helps to characterize the type of diffusion exhibited by the cells, providing insights into the stochastic aspects of their motion, namely whether the cells' movement is consistent with normal diffusion, subdiffusion, or superdiffusion.

Finally, we compute the velocities along the smoothed trajectories and use them as sparse samples to reconstruct the wound attractant field. This process involves solving a minimization problem for the vector components and lengths of the velocity field. The solution reduces to solving the Laplace equation with Dirichlet boundary conditions on the sparse samples and zero Neumann boundary conditions on the domain boundary. The result is a vector field where the direction and lengths of the vectors are interpolated/extrapolated from the Dirichlet conditions.

References

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