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Numerical methods for the controlled degenerate chemotaxis model in a weak formulation

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In this work, we study a controlled chemotaxis model using the finite element method. The model consists of a parabolic-parabolic system with degenerate diffusion and a control term representing the chemoattractant concentration. The goal is to determine an optimal control strategy to regulate the chemoattractant concentration, achieving a desired cell density and concentration, particularly in cancer treatment, where the objective is to limit cancer cells while minimizing effects on healthy tissues.

This is achieved by minimizing a quadratic problem where the cost function measures the errors between the cell density, the concentration, and the desired given states; the cost of the control is also minimized.

We propose a new approach that utilizes the concept of weak solutions with energy inequalities to be the set of admissible sets, unlike classical approaches that assume a strong regularity of state solutions. This approach ensures the existence of an optimal control while simplifying both the mathematical analysis and the adjoint system formulation, particularly in the case of degenerate diffusion.

In our recent work [1], we proposed a pioneering approach to address the challenge of analyzing weak solutions for the optimal control of chemotaxis models with a two-sided degenerate diffusion function. In this context, we prove the existence of solutions for both the direct and adjoint problems, despite the severe regularity issues induced by degeneracy.

The introduction of optimal control in such models presents numerical challenges. Indeed, handling optimal control in the weak solution framework remains particularly difficult. In the context of convection-diffusion equations with control, finite element methods have been employed to address control problems (see, for example, [2]). Additionally, in [3], the authors studied an optimal control problem for a two-dimensional attraction-repulsion chemotaxis.

In this work, we propose a novel numerical approach for optimal control of the isotropic degenerate Keller-Segel model using the finite elements method. Our goal is to numerically solve the optimal control problem associated with this parabolic PDE-constrained system.

The numerical resolution of such problems requires careful discretization and optimization strategies. Two main strategies exist: optimizing first or discretizing first, each leading to potentially different numerical results (see [4]).

We adopt the optimize-then-discretize approach, where we first derive the optimality conditions in the continuous model before discretizing and solving them. We begin by proving the existence and uniqueness of the discrete solution for the direct problem, followed by the adjoint problem.

For optimization problem, we use the gradient method, a first-order algorithm that relies on the cost function and its gradient at each step. Solving PDE-constrained optimal control problems remains challenging, requiring the storage of state, adjoint, and control variables at each time step.

Finally, we present numerical tests to assess the efficiency and stability of the gradient algorithm.

Keywords: chemotaxis, finite element method, optimal control, algorithm of gradient

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