Mathematical Methods and Models in Biosciences
June 15–20, 2025, Sofia, Bulgaria
https://biomath.math.bas.bg/biomath/index.php/bmcs

Reinfection induced multistability in an epidemic model

Pierre-Alexandre Bliman, Marcel Fang

Sorbonne Université, Université Paris Cité, CNRS, Inria, Laboratoire Jacques-Louis Lions, LJLL, EPC MUSCLEES, France
pierre-alexandre.bliman@inria.fr
marcel.fang@inria.fr

We consider in this presentation the effects induced on the transmission of a disease with imperfect immunity, by differences between primary and subsequent infections (i.e. reinfections), due e.g. to differential susceptibility or differential infectiousness. To this end, an 8-dimensional *two-stage* SEIRS reinfection model that extends the classical SEIRS model [1] is proposed, in which the parameters characteristic of the disease dynamics are different between primary infections and secondary infections.

The number of steady states of this model is assessed, depending upon the parameter values. It is shown that the reinfection induced heterogeneity may cause up to two endemic equilibria when the basic reproduction number of the model is less than one, and up to three endemic equilibria when it is greater than one. This suggests that, according to the parameter values, the model may present backward bifurcation, a feature quite important from the point of view of disease control. Simulations confirm this situation. The persistence of the disease is also examined.

Finally, we turn our attention to the study of several models with partial immunity, which can be considered as particular cases of the previous one. Based on a geometric approach developed by Li and Muldowney [2], it is shown that every trajectory of these simpler models converges asymptotically to an equilibrium, which depends upon the initial condition in case of multistability. More details may be found in [3].

Keywords: compartmental models in epidemiology, reinfection models, multistability, backward bifurcation, persistence, Li and Muldowney theory

References

[1] K. Shea, O. N. Bjørnstad, M. Krzywinski, Naomi Altman, Uncertainty and the management of epidemics, *Nature Methods*, 17:867–868, 2020.

[2] M. Y. Li, J. S. Muldowney, A Geometric Approach to Global-Stability Problems, *SIAM Journal on Mathematical Analysis*, 27:1070–1083, 1996.

[3] M. Fang, *Mathematical modelling, observation and identification of epidemiological models with reinfection*, PhD thesis, Sorbonne Université, 2024.