Figures of Cramér-von Mises statistic for continuous distributions

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A general procedure to generate samples from continuous standard uniform distribution has been described in [1]. The link between any order statistic and the uniform distribution is obtained via the cumulative distribution function. An experiment has been set to obtain a model for the Cramér-von Mises statistic as a function of sample size and cumulative probability.

The Cramér-von Mises test is used successfully in biological sciences (see [2], [3], and [4]), for comparing observed distributions of biological phenomena with theoretical models.

Here, following [5], based on the generated data, for continuous distributions, a series of figures of the Cramér-von Mises statistic as a function of the cumulative probability for some values of the sample size, as well as a series of figures of the Cramér-von Mises statistic as a function of the sample size for some values of the cumulative probability were obtained and their shape of dependence is discussed.

The result obtained by gradually increasing the sample size, shows us that the Cramér-von Mises statistic as a function of probability exhibits changes that ultimately lead to steeper tails. If we use the logarithmic scale of the representation for the Cramér-von Mises statistic, then we observe that with increasing sample size, the shape tends to symmetry (S_2 point symmetry group).

Looking at the dependencies of Cramér-von Mises statistic as function of the sample size for selected values of the cumulative probability one should notice that the most of the variation in the tails is for small n values. Even if it is with a slower variation at the median, it is still with a fast convergence, being approximated by a power function ($0.119n^{0.283n^{-1.662}}$).

Keywords: Cramér-von Mises criterion, goodness-of-fit tests
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