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Mathematical modelling of the neuromuscular activity of a motor unit

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The excitability of α-motoneurons and muscle cells and their corresponding behaviour is essential for the understanding of muscle contraction and neuromuscular diseases. Mathematical models, capable of accurately simulating muscle contraction, are very useful not only for the better understanding of the underlying processes, but also for numerous bio- and biomedical engineering applications. However, the precise mathematical description even of a single motor unit (MU) twitch, i.e., a single contraction of the MU—the basic functional unit in the skeletal muscle, has shown to be a very challenging task. To the best of the authors' knowledge there are no models describing the underlying mechanisms of the process, which have been fitted with a high level of accuracy to experimental data for a MU twitch.

In the present work, we present an integrated mathematical model of the neuromuscular activity of a motor unit. It is described in terms of ordinary differential equations and couples the models of Izhikevich of neural activity, the Williams model of calcium activity in the muscle fiber, and a Hill-type model of the resultant muscle force. We have introduced an important novelty in the coupling of the latter two, by introducing a sigmoid activation function, which significantly improves the descriptive capabilities of the model from a quantitative point of view. We have validated the model by fitting with a high degree of accuracy experimental data for twitches of nine different motor units (slow, fast fatigable, fast fatigue-resistant)—a result, which is not known in the scientific literature with a descriptive model. We also study numerically and show how the model parameters affect the model solution.

As a result, the present work provides a computational framework, under which one can perform computer simulations, or process and analyze real experimental data of neuromuscular activity and in the same time relate the findings to main characteristics of the underlying processes. We believe that at present such a framework is missing, but very much needed, in order to computationally study and/or obtain valuable hypotheses for many aspects of the motor unit activity, such as, e.g., neuromuscular disorders.