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Ecological resilience in microbial species: A multi-species cross-diffusion model for microbial competition and commensalism

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Microbial communities play a crucial role in ecological and biotechnological processes, but their dynamics is not fully understood. In this study, we introduce a novel multispecies cross-diffusion system to model ecological interactions – such as competition, commensalism, and neutralism – between microbial species. This PDE framework is motivated by an individual-based model (IBM) [1], and offers significantly higher computational efficiency and analytical tractability compared to the IBM.

We employ both analytical tools and numerical simulations to investigate the behaviour of the microbial community. In particular, we explore the resilience of bacteria with hypothetical metabolic capabilities in diverse ecological settings, comparing their fitness to species with well-characterised metabolic strategies. Although in nutrient-rich environments, faster-growing bacteria usually outcompete others, this is not necessarily the case in resource-limited settings. Some bacteria follow a yield strategy: growing more slowly, but using nutrients more efficiently than their fast-growing counterparts. In our model, we incorporate a growth rate versussussus yield trade-off and find that in nutrient-poor conditions, growth strategists initially accumulate biomass more rapidly, but over longer timescales, yield strategists ultimately dominate.

Our model successfully reconstructs the microbial interaction scenarios originally formulated in the individual-based model, including the the species involved in nitrification. Furthermore, we analyse the radial expansion of microbial populations by extracting travelling wave speeds in our system. This work provides a new mathematical framework for understanding microbial interactions and predicting their long-term dynamics, with implications for biotechnology and environmental conservation.

References

[1] E. Martinez-Rabert, C. van Amstel, C. Smith, W. T. Sloan, R. Gonzalez-Cabaleiro, Environmental and ecological controls of the spatial distribution of microbial populations in aggregates, *PLOS Computational Biology*, 18:e1010807, 2022.