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Increased competition reduces critical times for diffusing species

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We considered the spatiotemporal interaction between superior and inferior species, modelled by diffusive Lotka-Volterra competition system. The asymptotic steady-state solution of such system in isolated territory is the competitive exclusion, hence survival, of the superior species. This means that, starting from any initial distributions of the superior and inferior species, the inferior species will be driven to extinction, while the superior will dominate the territory. However, the knowledge that an infinite amount of time is needed to reach the total extinction-domination configuration is less practical for applications. Thus, the "critical time"—the finite time required for each species to be close enough to its steady state, whether total extinction or domination—has been studied quite recently.

In this study, we show that the critical times vary depending on the species' initial distributions. Since there are an infinite number of initial distributions a species can have, and due to the difficulty of deriving analytical expressions of the model solutions, we tested a few types of initial distributions numerically. Based on the correspondence between the transient solutions and the critical times, we found that the critical times of both the superior and inferior species decrease when the number or the width of the superior cluster is increased, due to the increase in the competition "sites" in the territory. On the other hand, in a sufficiently sparse territory, increasing the distance between the superior and inferior clusters increases the critical times. It is because the distance delays the competition and allows species to grow stronger before they start competing.

All the observations mentioned above show that the increased competition between superior and inferior species reduces the critical times. This work can be useful, for example, in developing spatiotemporal continuous models to investigate the optimal treatment for tumors, which have mainly been studied using discrete models.

Keywords: reaction-diffusion, critical time, steady state
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References

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