Mathematical Methods and Models in Biosciences
June 15–20, 2025, Sofia, Bulgaria
https://biomath.math.bas.bg/biomath/index.php/bmcs

Numerical analysis of a Keller–Segel model describing immune cell dynamics

Georgi Bazlyankov[1], Tihomir Ivanov[1]

[1]Faculty of Mathematics and Informatics,
Sofia University St. Kliment Ohridski, Bulgaria
gbazljanko@fmi.uni-sofia.bg
tbivanov@uni-sofia.bg

We study a Keller–Segel type partial differential equation model with logistic term and a flux-limiting chemotactic sensitivity function. The model aims to recreate the dynamics of immune cells after entering the central nervous system in the presence of a chemoattractant, as is the case in diseases such as multiple sclerosis.

We study traveling wave-type solutions as an intermediate asymptotic of the system as well as the different modes of pattern formation. Numerical experiments for the system are conducted, and the obtained solutions are compared to theoretical results derived for similar models such as special cases of Keller–Segel systems and the Fisher–KPP equation.

We derive a necessary condition for instability of the non-trivial steady state using Turing instability analysis. The numerically obtained solutions are used to illustrate the different modes of pattern formation—static, periodic, and irregular.

Keywords: immunology, partial differential equations, numerical analysis, traveling wave solutions, pattern formation

References

[1] R.B. Salako, S. Wenxian, "On traveling wave solutions in full parabolic Keller-Segel chemotaxis systems with logistic source.", *arXiv preprint*, arXiv:1901.02727 (2019).

[2] V.Calvez, B Perthame, S Yasuda, "Traveling wave and aggregation in a flux-limited Keller-Segel model.", *arXiv preprint*, arXiv:1709.07296 (2017).

[3] M. C. Lombardo, et al., "Demyelination patterns in a mathematical model of multiple sclerosis.", *Journal of mathematical biology*, 75 (2017): 373-417.