Modelling of activator-inhibitor dynamics via nonlocal integral operators

Roumen Anguelov, Stephanus Marnus Stoltz
Department of Mathematics and Applied Mathematics, University of Pretoria
roumen.anguelov@up.ac.za, stoltzstep@gmail.com

The theory of pattern formation through local self-activation and long range inhibition has been shown to account for much of the observed pattern forming regulatory phenomena [2]. This mechanism is captured mathematically by considering two species, activator and inhibitor, with different spatial diffusivity, so that the resulting model is a system of reaction diffusion equations. The formation of patterns occurring in such systems under certain conditions was discovered by Alan Turing in 1952. Independently of Turing's work, Gierer and Meinhard derived in 1972 their *Theory of Biological Pattern Formation* showing that patterns occur only if local self-enhancing reaction is coupled with an antagonistic reaction of long range [2,4]. The theory was embedded in a model comprising a system of reaction diffusion equations satisfying the Turing conditions. This model is now known as the Gierer-Meinhard model. It is used as a mathematical model for pattern formation in many different settings. For example, the Brusselator model for trimolecular chemical reactions is a particular case of it [1].

In this talk we propose modelling of the activation-inhibition mechanism of pattern formation by using nonlocal integral operators. This approach was pioneered in [3] for modelling of vegetation patterns. It turns out that the short range of the activation and the long range of the inhibition can be adequately represented via the supports of the kernels of the respective integrals. An advantage of using the nonlocal operator model from the point of view of its theoretical and numerical analysis is that it does not require smoothness of the solution with respect to the spatial variable. The applicability of this new approach is demonstrated on several biologically relevant examples.

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[2] A. Gierer, H. Meinhardt, *A theory of biological pattern formation*, Kybernetik **12** (1972) 30–39.

[3] R. Lefever, O. Lejeune, *On the origin of tiger bush*, Bulletin of Mathematical Biology **59** (1997) 263–294.

[4] H. Meinhardt, *Turings' theory of morphogenesis of 1952 and the subsequent discovery of the crucial role of local self-enhancement and long-range inhibition*, Interface Focus (2012), `http://dx.doi.org/10.1098/rsfs.2011.0097`.