

R_0 in continuous-time ecological models structured by age and space

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In this work we study the asymptotic behaviour in linear models of population dynamics by means of the basic reproduction number R_0 . Our aim is to give a practical approach to the computation of the reproduction number in continuous-time population models structured by age and/or space.

The traditional approach to the study of linear continuous-time population dynamics is the computation of the *Malthusian parameter*, i.e. the exponential growth rate of the population. Yet, another equivalent approach is possible which takes the generational viewpoint, [2], [3], [5]. Firstly, for each ecological model, one has to distinguish between birth terms and the other ones (e.g. mortality, transition). Then, in the homogeneous setting, the *basic reproduction number* is computed as the spectral radius of the next-generation operator. The classification into birth and non-birth terms is not uniquely determined and so, different interpretations of what a birth event is give rise to different expressions, [2] and [1]. Typically, the second approach provides more biological insight and results in an explicit expression allowing the study of evolutionary issues.

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