Review of a Predator-Prey Model with Two Limit Cycles

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The existence and the number of isolated periodic solutions (limit cycles) is one of the most delicate problems connected with two-dimensional predator-prey models. One of the first examples of a biological system modelling the interaction between prey and predators was formulated by Lotka in 1925 and independently by Volterra in 1927. It is a well-known that the Lotka-Volterra predator-prey model has a family of periodic orbits, but does not possess limit cycles and therefore the model is said to be structurally unstable. A biological cycle that persists over time must be insensitive to disturbances that might knock a population off its orbit. This means that as the disturbance dissipates it is expected for the populations to return to their original orbits. This is not the case, and the populations will remain in the new orbit unless another shock sends them onto yet another cycle. An ecologically stable cycle must therefore be isolated. For this reason the Lotka-Volterra model is not of much use in biological research, but since it is so elegant, it is still used in teaching examples. The Lotka-Volterra model is a special case of a much larger group: the quadratic population models and it can be shown that none of them can produce limit cycles. The surprising finding is that by combining two quadratic models we uncover a quadratic population model with two limit cycles. Although the model looks simple at first glance it provides a rich source of dynamics and deserves attention. In this paper we revisit a model that has its origin in the work of Dubois and Closset [1], based on the patchiness effect of marine plankton. A set of two quadratic population models interact as piecewise defined differential equations, which leads to interesting dynamics. The model has been discussed by Ren Yontai and Han Li [2] with some linguistic and typographic errors and provides an excellent vehicle for developing skills in mathematical modeling, differential equations and technology for the young researcher. We unpack the model and supplement the theory with rich graphical illustration. The compilation of the paper has the purpose of providing an example of how a young researcher, such as a post graduate student, can expand on an existing model by making use of current technology and also by investigating the effect of possible changes to the model.