Computational aspects of heat conduction models for biological tissues

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The idea that the heat equation is flawed due to the infinite speed of thermal propagation is not new. A hyperbolic model for heat conduction was proposed by Cattaneo in 1948 and Vernotte in 1958.

The application of femto-second lasers in many fields caused a rethink of the parabolic heat conduction model. In this talk we consider the paper by Kim and Guo in 2007 [1]. They developed a combined radiation and heat transfer model to simulate heat transfer in turbid tissues, covering three phases. After the irradiation phase they use the hyperbolic model to predict the temperature field. The authors quote several other papers to support their choice of the hyperbolic model; the arguments against the parabolic model is convincing. The numerical approximation is by finite differences with so called "error terms correction".

Van Rensburg and Sieberhagen [2] considered a number of papers on the numerical approximation of the hyperbolic heat transfer model where the authors used various methods to get rid of "spurious" oscillations. They pointed out that the oscillations were due to problems being ill-posed rather than the numerical method. However, a well-posed formulation required an extremely fine grid and was not practical. The authors did solve the problem for the one-dimensional case using FEM combined with d'Alembert's method.

We accept that the model in Kim and Guo is realistic. However, the model problem for the conduction phase is not well-posed due to the boundary conditions. We explain why, and also explain why the linearized dual phase lag hyperbolic model is better. Finally, we present results obtained with the mixed finite element method.

[1] K. Kim and Z. Guo, *Multi-time-scale heat transfer modeling of turbid tissues exposed to short-pulsed irradiations*, Computer methods and programs in Biomedicine, **86** (2007), 112-123.

[2] R.H. Sieberhagen and N.F.J. Van Rensburg, *Tracking a sharp crested wave front in hyperbolic heat transfer*, Applied Mathematical Modelling, **36** (2012), 33993410.