Codimension-2 saddle-node loop bifurcations as critical switch in neuronal excitability, filtering and susceptibility to stochastic ion channels

Jan-Hendrik Schleimer, Janina Hesse and Susanne Schreiber
Institute for theoretical Biology, Humboldt University, Berlin.
Bernstein Center for Computational Neuroscience Berlin.
jh.schleimer@hu-berlin.de, janina.hesse@bccn-berlin.de,
s.schreiber@biologie.hu-berlin.de

Keywords: Saddle-node loop bifurcation, neuronal excitability, ion channel stochasticity

Information processing in neurons is governed by the voltage dynamics of the cells' membrane. Filtering of synaptic stimuli and synchronisation to other neurons depends on the interplay of the bifurcation structure and the stochasticity of ion channels. Changes in filtering are organised by higher order bifurcations. We prove and derive conditions for the generic existence of a global codimension-2 saddle-node loop bifurcation in conductance-based neuronal models, like the Traub-Miles model. This bifurcation is reached by system parameters that affect the timescale separation between voltage and gating dynamics, such as temperature, capacitance or leak conductance [1]. The immediate implications for neuronal filtering are given by linear response theory of the phase reduction of the accompanying stochastic limit cycle [2]. The hallmark of the saddle-node loop transition is that, compared to the Bogdanov-Takens point, already infinitesimal parameter changes modify filtering. Around this critical point, we can furthermore prove that the tangent space to the isochrons [3] is spanned by the strongly stable manifold. This changes the systems susceptibility to different ion channel kinetics, suggesting that the impact of different noise sources in neurons can be derived from the bifurcation mediating the onset of spiking.

References

[1] J. Hesse, J.-H. Schleimer, S. Schreiber. *Qualitative changes in phase-response curve and synchronization at the saddle-node loop bifurcation.* Phys. Rev. E., accepted (2017).

[2] J.-H. Schleimer, M. Stemmler. Thor, *Coding of Information in Limit Cycle Oscillators*, Phys. Rev. Lett. **103** 248105 (2009).

[3] A. T. Winfree, *The Geometry of Biological Time*, Springer-Verlag (2001). **103** 248105.