Subcritical Bifurcation and Global Dynamics in an Imperfect Pulse Vaccination Model

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In this talk we investigate an SIVS epidemic model with imperfect vaccine, thus vaccinated individuals can also contract the infection. We consider pulse vaccination, that means we vaccinate fraction φ of population at times $t = nT$. The model reads as

$$\begin{cases} S'(t) &= \mu - \beta S(t)I(t) - \mu S(t) + \gamma I(t) + \theta V(t) \\ I'(t) &= \beta S(t)I(t) - (\mu+\gamma)I(t) + \sigma\beta V(t)I(t) \\ V'(t) &= -\sigma\beta V(t)I(t) - (\mu+\theta)V(t) \end{cases} \quad \text{if } t \neq nT,$$

$$\begin{cases} S\left(nT^+\right) &= (1-\varphi)S\left(nT^-\right) \\ I\left(nT^+\right) &= I\left(nT^-\right) \\ V\left(nT^+\right) &= V\left(nT^-\right) + \varphi S\left(nT^-\right) \end{cases} \quad \text{if } t = nT,$$

where $T > 0$ is fixed and $n \in \mathbb{Z}^+$.

It is known that in some vaccination models, backward bifurcation occurs and multiple subthreshold endemic equilibria exist, thus the behaviour of solution depends on the initial value [1]. We know that pulse vaccination can be more effective than constant vaccination, thus it is an interesting question to study whether backward bifurcation can arise in a pulse vaccination model.

We prove that backward bifurcation can occur in pulse vaccination model. First, we find a disease-free periodic solution, which is locally asymptotically stable in the whole phase space if $R_c < 1$. If $R_c > 1$, then the infection is strongly uniformly persistent in the population.

We perform the complete bifurcation analysis of a fixed point equation, where the most important tool was the Lyapunov-Schmidt method and we obtain a sufficient and necessary condition for the existence of subcritical bifurcation.

References

[1] F. Brauer, *Backward bifurcation in simple vaccination models*, J. Math. Anal. Appl. **298** 418–431 .