Biomath Communications

www.biomathforum.org/biomath/index.php/conference

A Note on the New Activation Function of Gompertz Type

Anton Iliev[1,2], Nikolay Kyurkchiev[1,2], Svetoslav Markov[2]

[1] Faculty of Mathematics and Informatics, University of Plovdiv Paisii Hilendarski, Plovdiv, Bulgaria

aii@uni-plovdiv.bg, nkyurk@uni-plovdiv.bg

[2] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria

smarkov@bio.bas.bg

Abstract In this note we construct a family of parametric Gompertz activation function (PGAF) based on hyperbolic tangent function.

We prove upper and lower estimates for the Hausdorff approximation of the sign function by means of this family.

Some comparisons between the hyperbolic tangent activation function and the new parametric Gompertz activation function are reported.

Numerical examples, illustrating our results are given.

Keywords: Parametric Gompertz activation function (PGAF), Sign function, Hausdorff distance, Upper and lower bounds

2010 Mathematics Subject Classification: 41A46, 62P10, 62M45

Citation: Anton Iliev, Nikolay Kyurkchiev, Svetoslav Markov, A Note on the New Activation Function of Gompertz Type,

http://dx.doi.org/10.11145/bmc.2017.10.201

1 Introduction

Sigmoidal functions (also known as “activation functions”) find multiple applications to neural networks [8]–[19], [55].

The modified hyperbolic tangent is a special S-shaped function constructed on the basis of the hyperbolic tangent function, which is expressed in terms of the exponent.

We study the distance between the sign function and a special class of activation functions, so-called parametric Gompertz activation function (PGAF).

The distance is measured in Hausdorff sense, which is natural in a situation when a sign function is involved. Precise upper and lower bounds for the Hausdorff distance are reported.

Any neural net element computes a linear combination of its input signals, and uses a logistic function to produce the result; often called “activation” function [20]– [22] .

2 Preliminaries

The following are common examples of activation functions:

a) logistic

$$\varphi_1(t) = \frac{1}{1+e^{-t}}; \tag{1}$$

b) Parametric Hyperbolic Tangent Activation (PHTA) function

$$\varphi_2(t) = \frac{e^{\beta t} - e^{-\beta t}}{e^{\beta t} + e^{-\beta t}} = 1 - \frac{2e^{-\beta t}}{e^{\beta t} + e^{-\beta t}}, \quad t \in \mathbb{R}, \quad \beta \geq 1; \tag{2}$$

c) Parametric Half Hyperbolic Tangent Activation (PHHTA) function

$$\varphi_3(t) = \frac{1-e^{-\beta t}}{1+e^{-\beta t}}, \quad t \in \mathbb{R}, \quad \beta \geq 1; \tag{3}$$

d) Parametric Fibonacci hyperbolic tangent activation function (FHTAF) [40] based on the Fibonacci hyperbolic tangent function [7]

$$\varphi_4(t) = \frac{\Psi^{\beta t} - \Psi^{-\beta t}}{\Psi^{\beta t} + \Psi^{-\beta t}}, \quad t \in \mathbb{R}, \quad \beta \geq 1; \tag{4}$$

where $\Psi = 1 + \phi = \frac{3+\sqrt{5}}{2} \approx 2.61$ and ϕ is the "Golden Section".

A survey of new mathematical models of Nature is presented based on the Golden Section and using a class of hyperbolic Fibonacci and Lucas functions in [6].

e) Parametric Soboleva' modified hyperbolic tangent activation function [41] based on Soboleva' modified hyperbolic tangent function [1]–[3]

$$\varphi_5(t) = m(t; c, d, c, d) = \frac{e^{ct} - e^{-dt}}{e^{ct} + e^{-dt}}. \tag{5}$$

The function finds application in approximating the current-voltage characteristics of light-emitting diodes [4].

In [23] the authors create the binary logistic regression model as to find the optimal vector $\beta = [\beta_0, \beta_1, \ldots, \beta_n]$ that best fits

$$y = \begin{cases} 1, & \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \epsilon > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Here, ϵ represents the error.

Evidently, in (1) t can be regarded as a variable, which is a linear weighted combination of independent variables $x = [x_1, \ldots, x_n]$ as

$$t \leftarrow \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n.$$

Thus, the binary logistic model is [23]:

$$F(x) = \frac{1}{1 + e^{-t(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n)}} \tag{6}$$

where $F(x)$ represents the probability of dependent variable $y = 1$.

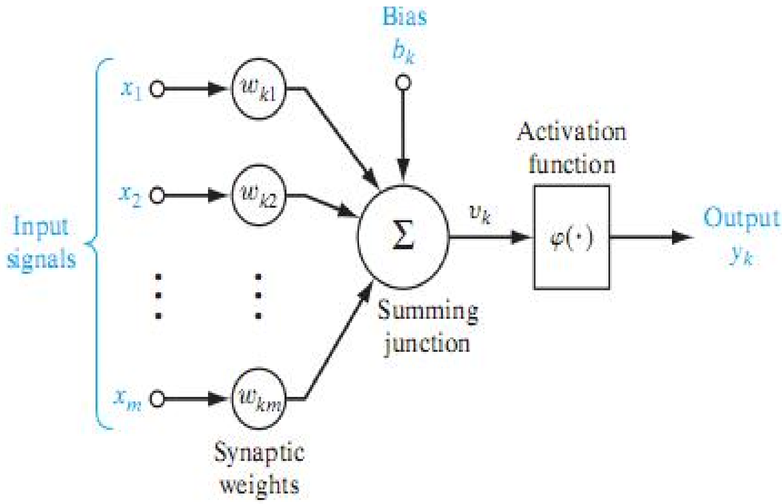

Figure 1: Nonlinear, parametrized function with restricted output range [5].

Training a multilayer perceptron with algorithms employing global search strategies has been an important research direction in the field of neural networks.

Multi–layer perceptrons are feed forward neural networks featuring universal approximation properties used both in regression problems.

The standard feed forward network with only a single hidden layer can approximate any continuous function uniformly on any compact set and any measurable function to any desired degree of accuracy [24]–[27], [5], [51].
For other results, see [56].

The nonlinear, parametrized function with restricted output range is visualized in Fig.1.

It is straightforward to extend this analysis to networks with multiple hidden layers.

For recurrent neural networks are typical:

a) stable outputs might be difficult to evaluate;

b) unexpected behavior (chaos, oscillation) might occur.

A survey of neural transfer activation functions can be found in [28].

Moreover, the nodes in the hidden layer are supposed to have a sigmoidal activation function which may be one of the following:

a) logistic sigmoid

$$\varphi_1(net) = \frac{1}{1+e^{-\beta\, net}}; \tag{7}$$

b) hyperbolic tangent

$$\varphi_2(net) = \frac{e^{\beta\, net} - e^{-\beta\, net}}{e^{\beta\, net} + e^{-\beta\, net}} \tag{8}$$

c) half hyperbolic tangent

$$\varphi_3(net) = \frac{1 - e^{-\beta\, net}}{1 + e^{-\beta\, net}} \tag{9}$$

d) Parametric Fibonacci hyperbolic tangent

$$\varphi_4(net) = \frac{\Psi^{\beta\, net} - \Psi^{-\beta\, net}}{\Psi^{\beta\, net} + \Psi^{-\beta\, net}} \tag{10}$$

e) Parametric Soboleva' modified hyperbolic tangent

$$\varphi_5(net) = \frac{e^{c\, net} - e^{-d\, net}}{e^{c\, net} + e^{-d\, net}} \tag{11}$$

where *net* denotes the input to a node and β, c and d are the slope parameters of the sigmoids.

A family of recurrence generating activation functions based on Gudermann function [53]

$$\begin{aligned} g_{i+1}(t) &= \tfrac{4}{\pi}\, arctg\left(e^{\frac{\pi}{2}(t+g_i(t))}\right) - 1;\ \ i = 0, 1, 2, \ldots, \\ g_0(t) &= \tfrac{4}{\pi}\, arctg\left(e^{\frac{\pi}{2}t}\right) - 1;\ \ g_0(0) = 0. \end{aligned}$$

is considered in [54].

Definition 1. *The sign function of a real number t is defined as follows:*

$$sgn(t) = \begin{cases} -1, & if \quad t < 0, \\ 0, & if \quad t = 0, \\ 1, & if \quad t > 0. \end{cases} \tag{12}$$

Definition 2. *[29], [30] The Hausdorff distance (the H–distance) [29] $\rho(f, g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,*

$$\rho(f, g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\}, \tag{13}$$

where $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t, x)|| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = max(|t_A - t_B|, |x_A - x_B|)$.

In [31]–[36], [40] the authors consider some families of recurrence generated parametric activation functions on the base of (7)–(11).

Definition 3. *The Gompertz function $\sigma_{\alpha,\beta}(t)$ is defined for α, $\beta > 0$ by [42]–[44]:*

$$\sigma_{\alpha,\beta}(t) = ae^{-\alpha e^{-\beta t}}, \tag{14}$$

where a is the upper asymptote when time approaches $+\infty$.

Gompertz functions are introduced by Benjamin Gompertz for the study of his demographic model, which represents a refinement of the Malthus model.

The functions find applications in modeling tumor growth and in population aging description.

In biology, the Gompertz curve or Gompertz function is commonly used to model growth process where the period of increasing growth is shorter than the period in which growth decreases.

For other results, see [45]–[50], [57].

3 Main Results

It is natural to define the following modified function:

f) New parametric Gompertz activation function (PGAF)

$$\varphi_6(t) = \frac{e^{-e^{-at}} - e^{-e^{at}}}{e^{-e^{-at}} + e^{-e^{at}}} \tag{15}$$

or

$$\varphi_6(net) = \frac{e^{-e^{-a\,net}} - e^{-e^{a\,net}}}{e^{-e^{-a\,net}} + e^{-e^{a\,net}}} \tag{16}$$

where *net* denotes the input to a node and a is the slope parameter of the sigmoid $\varphi_6(t)$.

In this Section we prove upper and lower estimates for the Hausdorff approximation of the sign function by means of $\varphi_6(t)$.

3.1 Approximation issues

The H-distance $d(sgn(t), \varphi_6(t))$ between the sgn function and the function φ_6 satisfies the relation:

$$\varphi_6(d) = \frac{e^{-e^{-ad}} - e^{-e^{ad}}}{e^{-e^{-ad}} + e^{-e^{ad}}} = 1 - d. \tag{17}$$

The following Theorem gives upper and lower bounds for d

Theorem 3.1. *For the Hausdorff distance d between the sgn function and the function φ_6 the following inequalities hold for*

$$a > \frac{1}{2}e^2 - 1$$

$$d_l = \frac{1}{2\,(1+a)} < d < \frac{\ln(2\,(1+a))}{2\,(1+a)} = d_r. \tag{18}$$

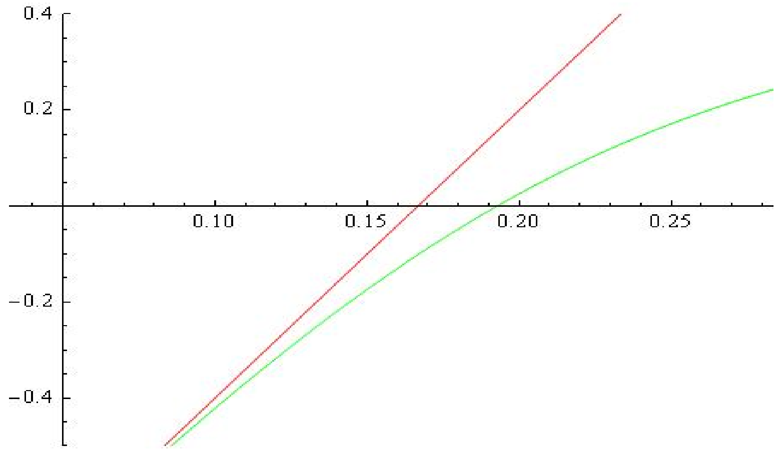

Figure 2: The functions $F(d)$ and $G(d)$ for $a = 5$.

Proof. We define the functions

$$F(d) = \frac{e^{-e^{-ad}} - e^{-e^{ad}}}{e^{-e^{-ad}} + e^{-e^{ad}}} - 1 + d \tag{19}$$

$$G(d) = -1 + (1 + a)d. \tag{20}$$

From the Taylor expansion we find (see Fig. 2)

$$F(d) - G(d) = O(d^3).$$

In addition $G'(d) > 0$ and for $a > \frac{1}{2}e^2 - 1$

$$G(d_l) < 0; \quad G(d_r) > 0.$$

This completes the proof of the inequalities (18).

Approximations of the $sgn(t)$ by (PGAF)–functions for various a are visualized on Fig. 3–Fig. 5.

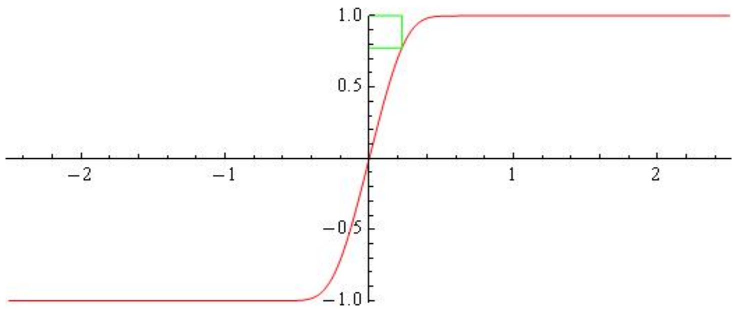

Figure 3: Approximation of the $sgn(t)$ by (PGAF) for $a = 4$; Hausdorff distance: $d = 0.225754$.

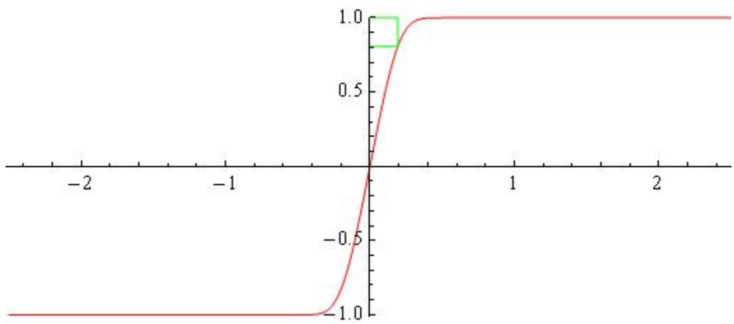

Figure 4: Approximation of the $sgn(t)$ by (PGAF) for $a = 5$; Hausdorff distance: $d = 0.192659$.

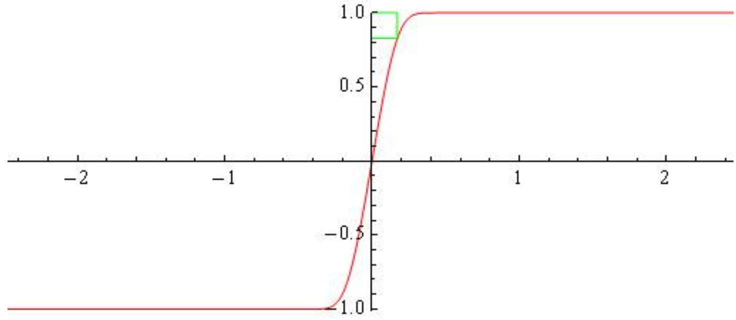

Figure 5: Approximation of the $sgn(t)$ by (PGAF) for $a = 5.83$; Hausdorff distance: $d = 0.172187$.

From the graphics it can be seen that the "saturation" is faster.

Some computational examples using relations (18) are presented in Table 1.

The last column of Table 1 contains the values of d computed by solving the nonlinear equation (17).

a	d_l	d_r	d from (17)
4	0.1	0.230259	0.225754
5	0.083333	0.207076	0.192659
5.83	0.0732064	0.191396	0.172187

Table 1: Bounds for d computed by (18) for various a.

From the above table, it can be seen that the right estimates for the value of the best Hausdorff distance (see (18)) are quite precise.

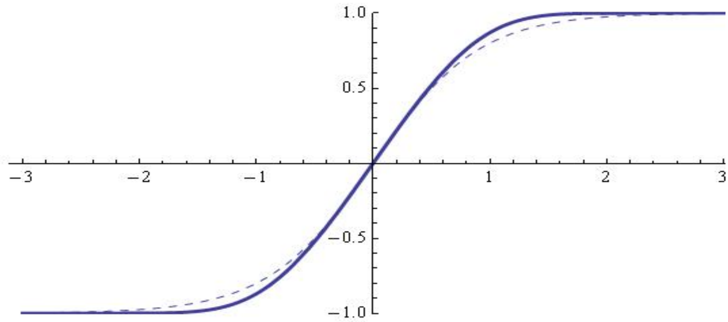

Figure 6: Comparison between the activation functions $\varphi_6(t)$ (thick) and $\varphi_2(t)$ (dashed) for fixed $a = 1.1$.

4 Comparison between the activation functions $\varphi_6(t)$ and $\varphi_2(t)$

Evidently (see Fig. 9)

$$\varphi_6(t) - \varphi_2(t) = \frac{a^3t^3}{6} + O(t^4).$$

Some comparison between the activation functions $\varphi_6(t)$ and $\varphi_2(t)$ for various a are visualized on Fig. 6–Fig. 8.

5 Conclusion

Hyperbolic tangent activation functions and their modifications with adaptive normalization play a useful role in neural network learning systems.

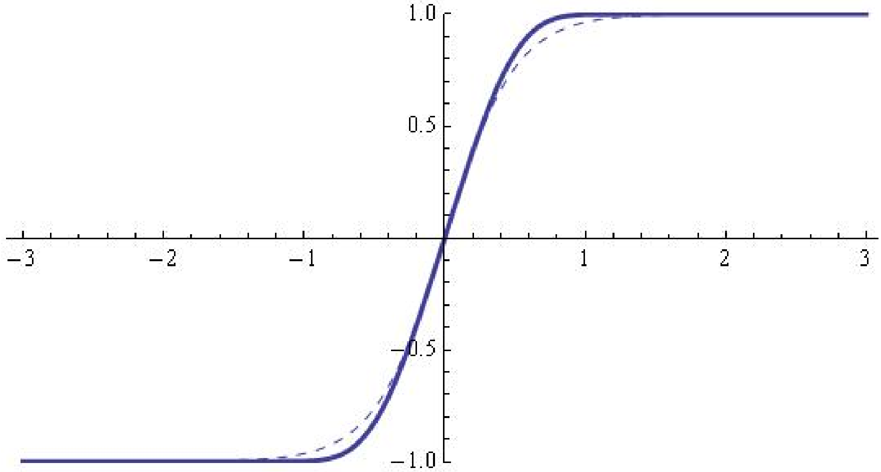

Figure 7: Comparison between the activation functions $\varphi_6(t)$ (thick) and $\varphi_2(t)$ (dashed) for fixed $a = 2$.

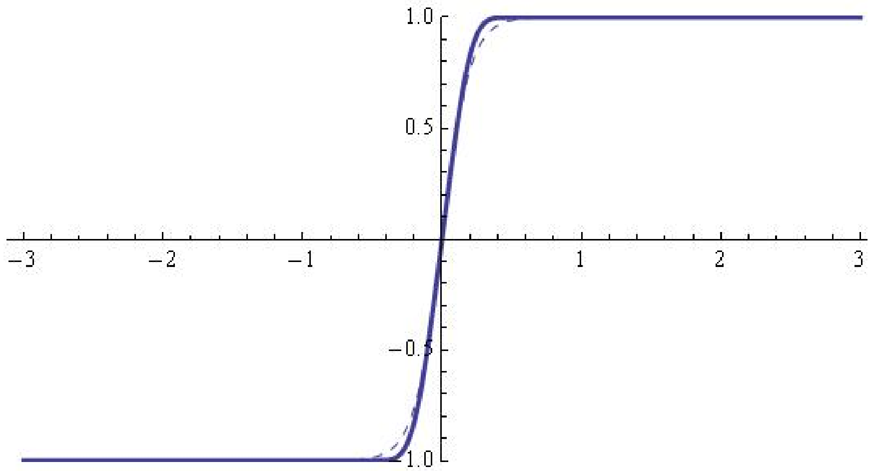

Figure 8: Comparison between the activation functions $\varphi_6(t)$ (thick) and $\varphi_2(t)$ (dashed) for fixed $a = 5$.

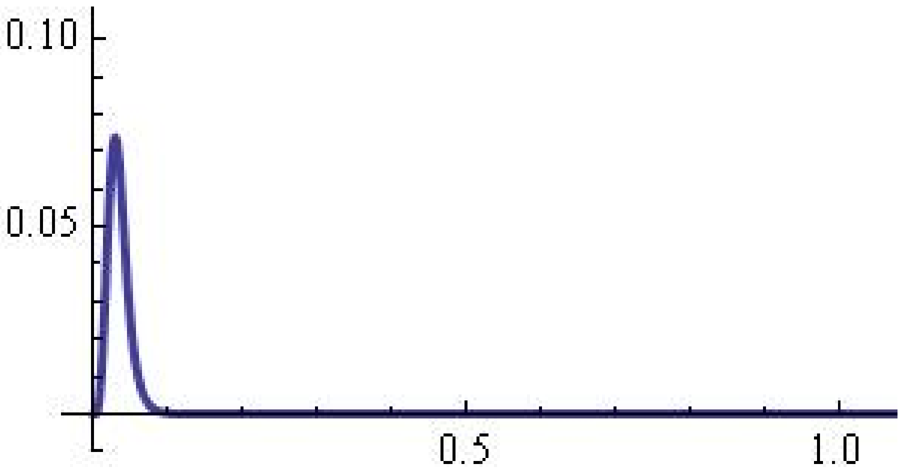

Figure 9: $\varphi_6(t) - \varphi_2(t) = \frac{a^3t^3}{6} + O(t^4)$.

A family of parametric Gompertz activation function (PGAF) based on hyperbolic tangent function is introduced in this paper it finds application in neural network theory and practice.

Theoretical and numerical results on the approximation in Hausdorff sense of the sgn function by means of functions belonging to the family are reported in the paper.

For other results, see [37]–[41].

Some techniques for recurrence generating of families of activation functions can be found in [52], [54].

Acknowledgments. This work has been supported by the project FP17–FMI–008 of Department for Scientific Research, Paisii Hilendarski University of Plovdiv.

The authors would like to thank the anonymous reviewers for their valuable comments and suggestions to improve the quality of the paper.

References

[1] E. V. Soboleva, V. V. Beskorovainyi, *The utility function in problems of structural optimization of distributed objects*, Kharkiv University of Air Force **121**, 2008.

[2] E. V. Soboleva, *The S-shaped utility function of individual criteria for multi-objective decision–making in design*, Kharkiv National University of Radioelectronics **247**, 2009.

[3] E. V. Soboleva, V. V. Beskorovainyi, *Identification of utility functions in multi–objective choice modelling by using S–shaped functions (PDF)*, Kharkiv National University of Radioelectronics, 2010, 50-54.

[4] V. I. Tuev, M. V. Uzhanin, *Using modified hyperbolic tangent function to approximate the current–voltage characteristics of field–effect transistors*, Tomsk Polytechnic University, 2009, 135-138.

[5] S. Haykin, Neural networks and learning machines, – 3rd ed., Copyright by Pearson Education, Inc., Upper Saddle River, New Jersey 07458, 2009.

[6] A. Stakhov, I. Tkachenko, *Hyperbolic Fibonacci Trigonometry*, Dokl. Akad. Nauk Ukrainy **7**, 1993, 9–14.

[7] Z. Trzaska, *On Fibonacci hyperbolic trigonometry and modified numerical triangles*, Fib. Quart. **34**, 1996, 129–138.

[8] N. Guliyev, V. Ismailov, *A single hidden layer feedforward network with only one neuron in the hidden layer can approximate any univariate function*, Neural Computation **28**, 2016, 1289–1304.

[9] D. Costarelli, R. Spigler, *Approximation results for neural network operators activated by sigmoidal functions*, Neural Networks **44**, 2013, 101–106.

[10] D. Costarelli, G. Vinti, *Pointwise and uniform approximation by multivariate neural network operators of the max-product type*, Neural Networks, 2016, http://dx.doi.org/10.1016/j.neunet.2016.06.002

[11] D. Costarelli, R. Spigler, *Solving numerically nonlinear systems of balance laws by multivariate sigmoidal functions approximation*, Computational and Applied Mathematics, 2016, http://dx.doi.org/10.1007/s40314-016-0334-8

[12] D. Costarelli, G. Vinti, *Convergence for a family of neural network operators in Orlicz spaces*, Mathematische Nachrichten, 2016, doi:10.1002/mana.20160006

[13] J. Dombi, Z. Gera, *The Approximation of Piecewise Linear Membership Functions and Lukasiewicz Operators*, Fuzzy Sets and Systems **154**(2), 2005, 275–286.

[14] I. A. Basheer, M. Hajmeer, it Artificial Neural Networks: Fundamentals, Computing, Design, and Application, Journal of Microbiological Methods **43**, 2000, 3–31.

[15] Z. Chen, F. Cao, *The Approximation Operators with Sigmoidal Functions*, Computers & Mathematics with Applications **58**, 2009, 758–765.

[16] Z. Chen, F. Cao, *The Construction and Approximation of a Class of Neural Networks Operators with Ramp Functions*, Journal of Computational Analysis and Applications **14**, 2012, 101–112.

[17] Z. Chen, F. Cao, J. Hu, *Approximation by Network Operators with Logistic Activation Functions*, Applied Mathematics and Computation **256**, 2015, 565–571.

[18] D. Costarelli, R. Spigler, *Constructive Approximation by Superposition of Sigmoidal Functions*, Anal. Theory Appl. **29**, 2013, 169–196.

[19] S. Lin, *Limitations of shallow nets approximation*, Neural Networks, 2017,
http://dx.doi.org/10.1016/j.neunet.2017.06.016

[20] D. Elliott, *A better activation function for artificial neural networks*, the National Science Foundation, Institute for Systems Research, Washington, DC, ISR Technical Rep. TR–6, 1993. Available: http://ufnalski.edu.pl/zne/ci_2014/papers/Elliott_TR_93-8.pdf

[21] G. Dallakyan, *Some Generalizations of Logistic Model*, Biomath Communications **4**, 2017, 15 pp.,
http://dx.doi.org/10.11145/bmc.2017.05.131

[22] K. Babu, D. Edla, *New algebraic activation function for multi-layered feed forward neural networks*, IETE Journal of Research, 2016, http://dx.doi.org/10.1080/03772063.2016.1240633

[23] S. Wang, T. Zhan, Y. Chen, Y. Zhang, M. Yang, H. Lu, H. Wang, B. Liu, P. Phillips, *Multiple Sclerosis Detection Based on Biorthogonal Wavelet Transform*, RBF Kernel Principal Component Analysis, and Logistic Regression, IEEE Access, Special section on advanced signal processing methods in medical imaging **4**, 2016, 7567–7576.

[24] G. Cybenko, *Approximation by superposition of a sigmoidal function*, Math. of Control Signals and Systems **2**, 1989, 303–314.

[25] K. Hornik, M. Stinchcombe, H. White, *Multi-layer feed forward networks are universal approximations*, Neural Networks **2**, 1989, 359–366.

[26] V. Kreinovich, O. Sirisaengtaksin, *3–layer neural networks are universal approximations for functionals and for control strategies*, Neural Parallel and Scientific Computations **1**, 1993, 325–346.

[27] H. White, *Connectionist nonparametric regression: multilayer feedforward networks can learn arbitrary mappings*, Neural Networks **3**, 1990, 535–549.

[28] W. Duch, N. Jankowski, *Survey of neural transfer functions*, Neural Computing Surveys **2**, 1999, 163–212.

[29] F. Hausdorff, Set Theory (2 ed.) (Chelsea Publ., New York, (1962 [1957]) (Republished by AMS-Chelsea 2005), ISBN: 978–0–821–83835–8.

[30] B. Sendov, Hausdorff Approximations, Kluwer, Boston, 1990, `http://dx.doi.org/10.1007/978-94-009-0673-0`

[31] N. Kyurkchiev, *A family of recurrence generated sigmoidal functions based on the Verhulst logistic function. Some approximation and modeling aspects*, Biomath Communications **3**(2), 2016, 18 pp.

[32] A. Iliev, N. Kyurkchiev, S. Markov, *A family of recurrence generated parametric activation functions with applications to neural networks*, International Journal on Research Innovations in Engineering Science and Technology **2**(1), 2017, 60–68.

[33] N. Kyurkchiev, S. Markov, *Hausdorff Approximation of the Sign Function by a Class of Parametric Activation Functions*, Biomath Communications **3**(2), 2016, 14 pp., `http://dx.doi.org/10.11145/bmc.2016.12.217`

[34] N. Kyurkchiev, A. Iliev, S. Markov, *Families of recurrence generated three and four parametric activation functions*, Int. J. Sci. Res. and Development **4**(12), 2017, 746–750.

[35] V. Kyurkchiev, N. Kyurkchiev, *A family of recurrence generated functions based oh Half-hyperbolic tangent activation functions*, Biomedical Statistics and Informatics **2**(3), 2017, 87–94.

[36] V. Kyurkchiev, A. Iliev, N. Kyurkchiev, *On some families of recurrence generated activation functions*, Int. J. of Sci. Eng. And Appl. Sci. **3**(3), 2017.

[37] N. Kyurkchiev, S. Markov, Sigmoid functions: Some Approximation and Modelling Aspects, LAP LAMBERT Academic Publishing, Saarbrucken, 2015, ISBN 978–3–659–76045–7.

[38] N. Kyurkchiev, S. Markov, *On the Hausdorff distance between the Heaviside step function and Verhulst logistic function*, J. Math. Chem. **54**(1), 2016, 109–119,
`http://dx.doi.org/10.1007/S10910-015-0552-0`

[39] N. Kyurkchiev, A. Iliev, *On the Hausdorff distance between the Heaviside function and some transmuted activation functions*, Mathematical Modelling and Applications **2**(1), 2016, 1–5.

[40] N. Kyurkchiev, A. Iliev, *A note on the new Fibonacci hyperbolic tangent activation function*, Int. J. of Innovative Science, Engineering and technology **4**(5), 2017.

[41] A. Golev, A. Iliev, N. Kyurkchiev, *A Note on the Soboleva' Modified Hyperbolic Tangent Activation Function*, International Journal of Innovative Science Engineering and Technology **4**(6), 2017.

[42] B. Gompertz, *On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of the life contingencies*, Philos. Trans. R. Soc. London, **115**, 1825, 513–585.

[43] M. A. Savageau, *Allometric morphogenesis of complex systems: A derivation of the basic equations from first principles*, Proceedings of the National Academy of Sciences, 1979, 5413–5417.

[44] Z. Bajzer, S. Vuk-Pavlovic, Mathematical Modeling of Tumor Growth Kinetics, In: Modeling and Simulation in Science, Engineering and Technology (J. Adam and N. Bellomo, Eds.), 84–133, Springer Science+Business Media, LLC, 1997.

[45] A. K. Laird, S. A. Tyler, A. D. Barton, *Dynamics of normal growth*, Growth **29**, 1965, 233–248.

[46] A. K. Laird, *Dynamics of growth in tumors in normal organisms*, National Cancer Institute Monograph, **30**, 1969, 15–28.

[47] C. P. Winsor, *The Gompertz curve as a growth curve*, Proc. Natl. Acad. Sci. USA **18**, 1932, 1–8.

[48] X. Xu, *The biological foundation of the Gompertz model*, Int. J. Biomed. Comput. **20**, 1987, 35–39.

[49] W. S. Kendal, *Gompertzian growth as a consequence of tumor heterogeneity*, Math. Biosci. **73**, 1985, 103–107.

[50] C. L. Frenzen, J. D. Murray, *A cell kinetics justification for Gompertz' equation*, SIAM J. Appl. Math. **46**, 1986, 614–629.

[51] R. Rojas, Neural Networks. A Systematic Introduction, Springer, 1996.

[52] N. Kyurkchiev, A. Iliev, S. Markov, Some techniques for recurrence generating of activation functions, LAP LAMBERT Academic Publishing, 2017, ISBN 978–3–330–33143–3.

[53] F. W. J. Olver, D. W. Lozier, R. F. Boisvert, C. W. Clark, NIST Digital Library of Mathematical Functions, Cambridge University Press, 2010, 951 pp.

[54] A. Malinova, A. Golev, A. Iliev, N. Kyurkchiev, *A Family of Recurrence Generating Activation Functions Based on Gudermann Function*, International Journal of Engineering Researches and Management Studies, **4**(8), 2017, 38–48.

[55] N. Gyliyev, V. Ismailov, *On the approximation by single hidden layer feed forward neural networks with fixed weights*, arXiv:1708.06219v1 [cs. NE] 21 August 2017.

[56] Y. Han, S. Etigowni, H. Li, S. Zonouz, A. Petropulu, *Watch Me, but Don't Touch Me! Contactless Control Flow Monitoring via Electromagnetic Emanations*, arXiv:1708.09099v1 [cs.CR] 30 Aug 2017.

[57] R. Anguelov, M. Borisov, A. Iliev, N. Kyurkchiev, S. Markov, *On the chemical meaning of some growth models possessing Gompertzian-type property*, Math. Meth. Appl. Sci., 2017, p. 1–12, `http://dx.doi.org/10.1002/mma.4539`