Mathematical Analysis of a Size Structured Tree-Grass Model in Savanna Ecosystems

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Keywords: savanna, modeling, qualitative analysis, numerical simulation

Several continuous competition-based models have been developed to study the possible coexistence of trees and grass in savanna ecosystems according to environmental parameters such as climate or fire regime. In those models fire intensity is a fixed parameter. Here, we model fire intensity through an increasing function of grass biomass and let fire return-time vary according to climate types. (For instance, in Africa, in moist savannas fire occurs every 1-2 years, while in dry savannas it is every 3-10 years.) Following [1], we also consider a tree-grass compartmental model that distinguishes small trees (like saplings) that are sensitive to fire from tall trees that are not.

On those bases, we model the savanna vegetation dynamics through three state variables (biomass of grass and of the two classes of woody plants) involved in a system of three interrelated, non-linear equations.

We carry out a qualitative analysis that highlights three ecological thresholds namely, the destruction ratio of trees due to causes which are different of fire, the survival ratio of grass after the "predation" of trees and the survival ratio of trees after fire. These thresholds summarize the dynamics of the system. Finally, we develop a non-standard numerical scheme [2], and show some numerical simulations to illustrate our analytical results.

References

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