Numerical Analysis in Size-Structured Population Models with Dynamical Environment

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We will study the numerical integration of a nonlinear model that describes the dynamics of size-structured populations feeding on a dynamical food-source. Such models are determined by the next equations:

$$u_t + (g(x,S(t))\,u)_x = -\mu(x,S(t))\,u, \quad 0 < x < x_M(t),\ t > 0, \tag{1}$$

$$g(0,S(t))\,u(0,t) = \int_0^{x_M(t)} \alpha(x,S(t))\,u(x,t)\,dx\,, \quad t > 0, \tag{2}$$

$$u(x,0) = \phi(x), \quad 0 \le x \le x_M(t), \tag{3}$$

$$S'(t) = f(t,S(t),I(t)), \quad t > 0, \qquad S(0) = s_0, \tag{4}$$

$$I(t) = \int_0^{x_M(t)} \gamma(x,S(t))\,u(x,t)\,dx, \quad t > 0. \tag{5}$$

where $x_M(0) = x_M$ and $x_M(t)$ represents the characteristic curve that begins at the maximum size. The integration of (1)-(5) is made by means of a suitable method for the coupled problem. We will analyse the consistency, stability and convergence properties of the numerical scheme. Special attention is devoted to the requirements imposed to the quadrature rule. Also, we will analyse the behaviour of such numerical scheme in the integration of the dynamics of *Daphnia magna*, feeding on a dynamical algal population [1].

References

[1] S.A.L.M. Kooijman & J.A.J. Metz, On the dynamics of chemically stressed populations: the deduction of population consequences from effects on individuals, *Ecotox. Env. Safety*, 8: 254–274, 1984.