Linear and Quasi-linear Spaces of Interval Maps

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Interval-valued functions, or briefly interval functions, are traditionally associated with numerical analysis and validated computing, see for instance [4, 5]. Such maps also occur naturally as mathematical tools for modeling phenomena where uncertainty is present, such as in biological dynamical systems [3]. Recently interval functions have also been applied to problems in pure mathematics, such as the Dedekind order completion of spaces of continuous functions [1]. In [2] the structure of a linear space was introduced on the set of finite Hausdorff continuous interval functions with domain an open subset of Euclidean space $\mathbb{R}^n$. In this paper we generalize this result to the case of functions defined on an arbitrary topological space. It should be noted that problem of defining algebraic operations on spaces of interval functions is nontrivial, due to the fact that the set of compact intervals in $\mathbb{R}$ is not a group with respect to the Minkowski sum.

References

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