Biomath Communications

www.biomathforum.org/biomath/index.php/conference

On a "Type I Half–logistic Modified Weibull" Model: Some Extended Models and Applications

Nikolay Kyurkchiev[1,2]

[1]Faculty of Mathematics and Informatics,
University of Plovdiv Paisii Hilendarski,
24 Tzar Asen Str., 4000 Plovdiv, Bulgaria,
e-mail: nkyurk@uni-plovdiv.bg,

[2]Institute of Mathematics and Informatics,
Bulgarian Academy of Sciences,
Acad. G. Bonchev Str., Bl. 8, 1113 Sofia, Bulgaria

Keywords: four parameter extended type I half–logistic modified Weibull (TIHLMW) distribution; 6 parameters G–family of cdf – $Q(t)$ with baseline cdf – $M(t)$; confidential curves; "super saturation"; Heaviside step–function; Hausdorff distance

Abstract

The cumulative distribution function (cdf) corresponding to the "four parameter extended type I half–logistic modified Weibull (TIHLMW) distribution" is [1]:

$$M(t) = \frac{1 - e^{-\lambda(\alpha_1 t + \theta t^{\beta_1})}}{1 + e^{-\lambda(\alpha_1 t + \theta t^{\beta_1})}},$$

Citation: Nikolay Kyurkchiev, On a "Type I Half–logistic Modified Weibull" Model: Some Extended Models and Applications, Biomath Communications 7, pp. 4-13, https://doi.org/10.11145/bmc.2020.03.029

where λ, θ, β_1 are positive shape parameters and α_1 is a scale parameter. Also of interest to the specialists is the task of approximating the Heaviside function $h_{t_0}(t)$ where t_0 is the "median" by the new cumulative function in the Hausdorff sense. Following the results given in [2] we will generate the "new 6–parameters G–family of cdf – $Q(t)$ with baseline cdf – $M(t)$":

$$Q(t) = e^{-\frac{2\alpha\beta e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}{1-e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}}\left(2 - e^{-\frac{2\beta e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}{1-e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}}\right)^{\alpha}$$

We also study the "saturation" by this family and "confidential curves" $Q_1(t)$ and $Q_2(t)$ for which $Q_1(t) \leq Q(t) \leq Q_2(t)$

Some numerical examples with real data from Biostatistics, Growth theory and Computer viruses propagation, using *CAS MATHEMATICA* illustrating our results are given.

It is shown that the study of the two characteristics - "confidential curves" and "super saturation" is a must when choosing the right model.

1 Introduction and Preliminaries

Definition 1. *Consider the following cumulative distribution function (cdf) corresponding to the "Four parameter extended type I half–logistic modified Weibull (TIHLMW) distribution" [1]:*

$$M(t) = \frac{1 - e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}{1 + e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}, \tag{1}$$

where λ, θ, β_1 are positive shape parameters and α_1 is a scale parameter.

Definition 2. *The shifted Heaviside step function is defined by*

$$h_{t_0}(t) = \begin{cases} 0, & \textit{if} \quad t < t_0, \\ [0,1], & \textit{if} \quad t = t_0, \\ 1, & \textit{if} \quad t > t_0 \end{cases} \tag{2}$$

Definition 3. *[3] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,*

$$\rho(f,g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = max(|t_A - t_B|, |x_A - x_B|)$.

Definition 4. *In [2] Bantan, Jamal, Chesneau and Elgarhy introduced a new power family of distributions with c.d.f.*

$$Q(t) = e^{\alpha\beta\left(1 - \frac{1}{G(t)}\right)} \left(2 - e^{\beta\left(1 - \frac{1}{G(t)}\right)}\right)^{\alpha} \tag{3}$$

where $\alpha, \beta \in R^+$ and $G(t)$ is a c.d.f. of a baseline continuous distribution. The following result shows some inequalities involving $Q(t)$ (see, Proposition 1 [2]):

$$e^{\alpha\beta\left(1 - \frac{1}{G(t)}\right)} \left(2 - G(t)^{\beta}\right)^{\alpha} \leq Q(t) \leq 2^{\alpha} e^{\alpha\beta\left(1 - \frac{1}{G(t)}\right)}. \tag{4}$$

In this paper we study some properties of the cdf – $M(t)$ and the family (3) with baseline cdf – $G(t) = M(t)$.

2 Main Results.

When studying the intrinsic properties of the families $M(t)$ and $Q(t)$ (with baseline cdf – $M(t)$), it is also appropriate to study the "saturation" to the horizontal asymptote.

2.1 The cdf $M(t)$.

In this Section we study the one–sided Hausdorff approximation of the Heaviside step–function $h_{t_0}(t)$ by means of family (1).

Let t_0 is a positive root of the nonlinear equation $M(t_0) - \frac{1}{2} = 0$.

The one–sided Hausdorff distance d satisfies the relation

$$M(t_0 + d) = \frac{1 - e^{-\lambda(\alpha_1(t_0+d)+\theta(t_0+d)^{\beta_1})}}{1 + e^{-\lambda(\alpha_1(t_0+d)+\theta(t_0+d)^{\beta_1})}} = 1 - d. \qquad (5)$$

We illustrate the "saturation" with the cdf (1) for various α_1, β_1, λ, θ (see, Fig. 1)

2.1.1 Some Applications.

Example 1. Here we will present a new analysis of Conficker propagation in 2008 and we explore the Network Telescope project's daily dataset [4], [5] collected on November 21, 2008.

We analyze the following data

$data\_Conficker :=$

$\{\{0.1, 10\}, \{1, 150\}, \{2, 300\}, \{3, 600\}, \{4, 2500\}, \{5, 5000\},$

$\{6, 7500\}, \{7, 13000\}, \{8, 19000\}, \{9, 25000\}, \{10, 31000\},$

$\{11, 37000\}, \{12, 44000\}, \{13, 52000\}, \{14, 58000\}, \{15, 66000\},$

$\{16, 74000\}, \{17, 81000\}, \{18, 86000\}, \{19, 89000\}, \{20, 92000\},$

$\{21, 92500\}\}$

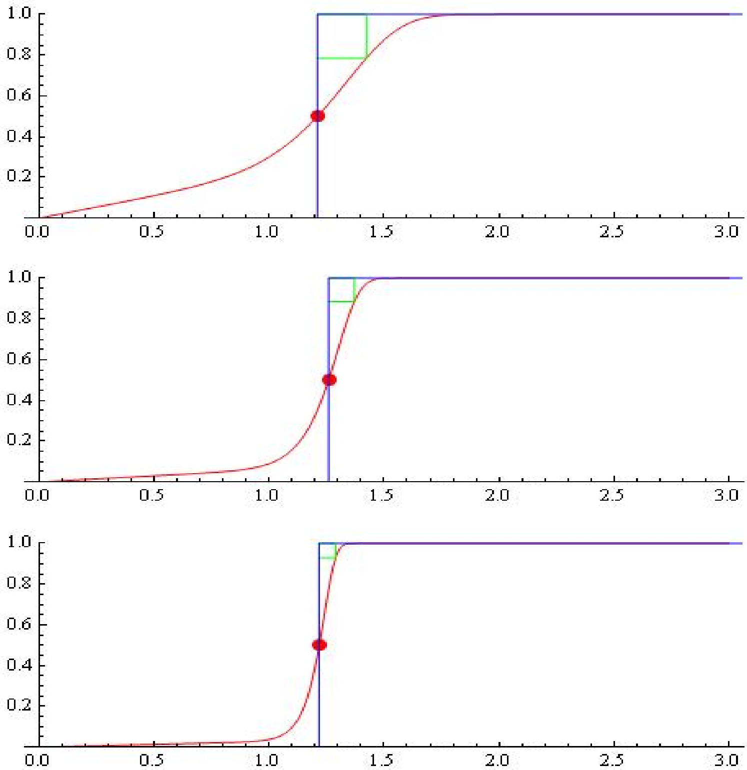

Figure 1: a) $\alpha_1 = 2.2$, $\beta_1 = 6$, $\lambda = 0.2$, $\theta = 0.9$; $t_0 = 1.2104$; Hausdorff distance $d = 0.2133056$; b) $\alpha_1 = 1.2$, $\beta_1 = 12$, $\lambda = 0.1$, $\theta = 0.6$; $t_0 = 1.25856$; Hausdorff distance $d = 0.112747$; c) $\alpha_1 = 1.1$, $\beta_1 = 20$, $\lambda = 0.05$, $\theta = 0.4$; $t_0 = 1.21794$; Hausdorff distance $d = 0.0714225$.

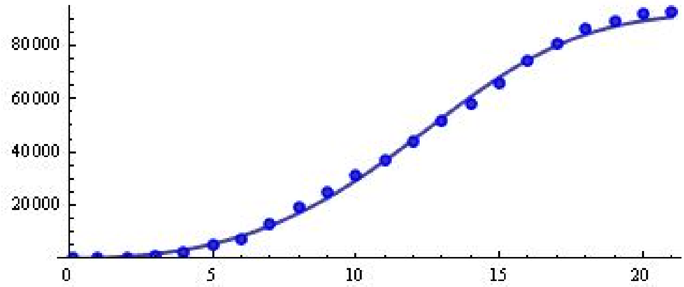

Figure 2: The fitted model $M^*(t)$.

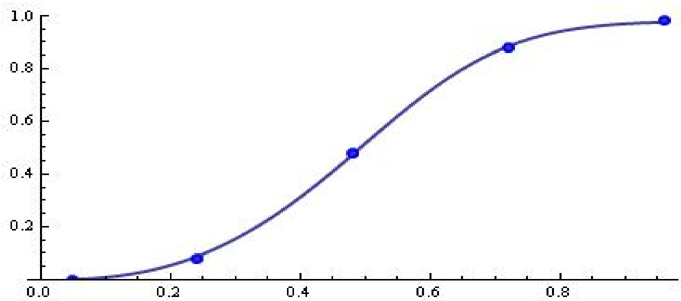

Figure 3: The fitted model $M^*(t)$.

The $M^*(t) = \omega M(t)$ for $\omega = 92500$; $\alpha_1 = 0.05$; $\beta_1 = 2.69138$; $\lambda = 0.1$ and $\theta = 0.0123854$ is visualized on Fig. 2.

Example 2. We analyze the data given in [6].

The $M^*(t) = \omega M(t)$ for $\omega = 0.98$; $\alpha_1 = 0.01$; $\beta_1 = 2.55582$; $\lambda = 0.4136371$ and $\theta = 16,6566$ is visualized on Fig. 3.

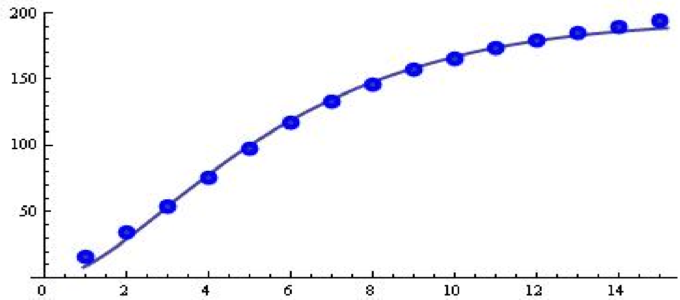

Figure 4: The fitted model $Q^*(t)$.

2.2 Some properties of the family (3) with baseline cdf $M(t)$.

Formally, we will generate the "new" cdf – $Q(t)$ with baseline cdf – $M(t)$:

$$Q(t) = e^{-\frac{2\alpha\beta e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}{1-e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}} \left(2 - e^{-\frac{2\beta e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}{1-e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}}\right)^{\alpha} \tag{6}$$

Example 3. We examine the data for the growth of red abalone Haliotis Rufescens in Northern California [7].

For this data the fitted model $Q^*(t) = \omega Q(t)$ for $\omega = 194$; $\alpha_1 = 0.6$; $\beta_1 = 0.2$; $\lambda = 0.361$; $\theta = 2.3$; $\alpha = 0.0496998$ and $\beta = 57.7664$ is visualized on Fig. 4.

We study the Hausdorff approximation of the Heaviside step function $h_{t_0}(t)$ where t_0 is the "median" by family of type (6).

Following the ideas given in [2] we find the two–sided bounds:

$$Q_1(t) \le Q(t) \le Q_2(t) \tag{7}$$

where

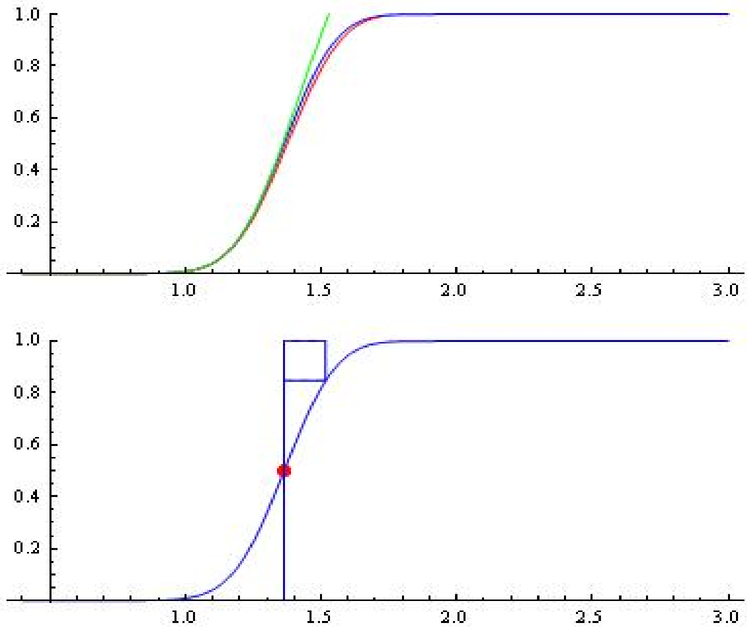

Figure 5: a) The two-sided bounds (7) for $\alpha_1 = 1.2$; $\beta_1 = 7$; $\lambda = 0.1$; $\theta = 0.6$ and $\alpha = 0.5$; $\beta = 1$; b) The model $Q(t)$ for $\alpha_1 = 1.2$; $\beta_1 = 7$; $\lambda = 0.1$; $\theta = 0.6$ and $\alpha = 0.5$; $\beta = 1$; H–distance $d = 0.153005$
.

$$Q_1(t) = e^{-\frac{2\alpha\beta e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}{1-e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}} \left(2 - \left(\frac{1-e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}{1+e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}\right)^{\beta}\right)^{\alpha},$$

$$Q_2(t) = 2^{\alpha} e^{-\frac{2\alpha\beta e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}{1-e^{-\lambda(\alpha_1 t+\theta t^{\beta_1})}}}.$$

The obtained two-sided estimations in particular case for $\alpha_1 = 1.2$; $\beta_1 = 7$; $\lambda = 0.1$; $\theta = 0.6$ and $\alpha = 0.5$; $\beta = 1$ are given in Fig. 5 a.

Let t_0 is the value for which $Q(t_0) = \frac{1}{2}$.

The Hausdorff distance d between the function $h_{t_0}(t)$ and $Q(t)$ satisfies the relation

$$Q(t_0 + d) = 1 - d. \quad (8)$$

For fixed $\alpha_1 = 1.2$; $\beta_1 = 7$; $\lambda = 0.1$; $\theta = 0.6$ and $\alpha = 0.5$; $\beta = 1$ from the nonlinear equation (8) we have $d = 0.153005$ (see, Fig. 5 b).

2.3 Concluding remarks.

The results obtained in this article can be successfully continued for generating of some new models based on known in literature "G-families of cumulative distribution functions".

The new model (6) has been applied widely in life testing experiments.

From Fig. 5 it can be seen that these estimations can be used as "confidence bounds", which are extremely useful for the specialists in the choice of model for cumulative data approximating in areas of Biostatistics, Population dynamics, Growth theory, Debugging and Test theory, Computer viruses propagation, Financial and Insurance mathematics.

Exploring both features - "confidential curves" and "super saturation" is a must when choosing the right model.

For other results, see [8]–[35].

Acknowledgment

This work has been accomplished with the financial support by the Grant NoBG05M2OP001-1.001-0003, financed by the Science and Education for Smart Growth Operational Program (2014-2020) and co-financed by the European Union through the European structural and Investment funds.

References

[1] I. Elbatal, M. Zayad, M. Rasekhi, A. Afify, Z. Iqbal, A new extended Weibull model for lifetime data, *J. of Appl. Prob. and Stat,*, **4**, No 1 (2019), 57–73.

[2] R. Bantan, F. Jamal, Ch. Chesneau, M. Elgarhy, A New Power Topp-Leone Generated Family of Distributions with Applications, *Entropy* (2019).

[3] B. Sendov, *Hausdorff Approximations*, Kluwer, Boston (1990).

[4] C. Zhang, S. Zhou, B. Chain, Hybrid Epidemics–A Case Study on Computer Worm Conficker, *PLoS ONE*, **10**, No. 5 (2015), e0127478.

[5] P. Porras, H. Saidi, V. Yegneswaran, An Analysis of Conficker's Logic and Rendezvous Points, SRI international technical report, March 19, (2009).

[6] N. Kyurkchiev, S. Markov, *Sigmoid functions: Some Approximation and Modelling Aspects*, LAP LAMBERT Academic Publishing, Saarbrucken (2015), ISBN 978-3-659-76045-7.

[7] L. RogersBennett, D. W. Rogers, S. A. Schultz, Modeling growth and mortality of red abalone Haliotis Rufescens in Northern California, J. of Shellfish Research, **26** (3) (2007), 719-727.

[8] A. Afify, A. Abdellatif, The extended Burr XII distribution: properties and applications, *J. of Nonlinear Sci. and Appl.*, **13** (2020), 133–146.

[9] S. Markov, Reaction networks reveal new links between Gompertz and Verhulst growth functions, *Biomath*, **8**, No. 1 (2019).

[10] M. Tahir, G. Cordeiro, Compounding of distributions: A survey and new generalized classes, *J. of Stat. Distr. and Appl.,* **3**, No. 13 (2016), 1–35.

[11] N. Kyurkchiev, On a sigmoidal growth function generated by reaction networks. Some extensions and applications, *Communications in Applied Analysis*, **23**, No. 3 (2019), 383–400.

[12] Kyurkchiev N., S. Markov, On the Hausdorff distance between the Heaviside step function and Verhulst logistic function, *J. Math. Chem.*, **54**, No. 1 (2016), 109–119.

[13] N. Kyurkchiev, A. Iliev, S. Markov, *Some Techniques for Recurrence Generating of Activation Functions: Some Modeling and Approximation Aspects*, LAP LAMBERT Academic Publishing (2017), ISBN: 978-3-330-33143-3.

[14] A. Iliev, N. Kyurkchiev, S. Markov, A Note on the New Activation Function of Gompertz Type, *Biomath Communications*, **4**, No. 2 (2017), 20 pp.

[15] R. Anguelov, M. Borisov, A. Iliev, N. Kyurkchiev, S. Markov, On the chemical meaning of some growth models possessing Gompertzian-type property, *Math. Meth. Appl. Sci.*, (2017), 1–12.

[16] R. Anguelov, N. Kyurkchiev, S. Markov, Some properties of the Blumberg's hyper-log-logistic curve, *BIOMATH*, **7**, No. 1 (2018), 8 pp.

[17] S. Markov, A. Iliev, A. Rahnev, N. Kyurkchiev, On the exponential–generalized extended Compertz cumulative sigmoid, *International Journal of Pure and Applied Mathematics*, **120**, No. 4 (2018), 555–562.

[18] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Some software reliability models: Approximation and modeling aspects*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-82805-0.

[19] A. Iliev, N. Kyurkchiev, A. Rahnev, T. Terzieva, *Some models in the theory of computer viruses propagation*, LAP LAMBERT Academic Publishing (2019), ISBN: 978-620-0-00826-8.

[20] A. Iliev, N. Kyurkchiev, S. Markov, On the approximation of the cut and step functions by logistic and Gompertz functions, *Biomath*, **4** (2015), 2–13.

[21] N. Kyurkchiev, A. Iliev, A. Rahnev, A new class of activation functions based on the correcting amendments of Gompertz-Makeham type, *Dynamic Systems and Applications*, **28**, No. 2 (2019), 243–257.

[22] N. Kyurkchiev, A. Iliev, A. Rahnev, *Some Families of Sigmoid Functions: Applications to Growth Theory*, LAP LAMBERT Academic Publishing (2019), ISBN: 978-613-9-45608-6.

[23] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Nontrivial Models in Debugging Theory (Part 2)*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-87794-2.

[24] A. Malinova, O. Rahneva, A. Golev, V. Kyurkchiev, Investigations on the Odd-Burr-III-Weibull cumulative sigmoid. Some applications, *Neural, Parallel, and Scientific Computations*, **27**, No. 1 (2019), 35–44.

[25] N. Kyurkchiev, Investigations on a hyper-logistic model. Some applications, *Dynamic Systems and Applications*, **28**, No. 2 (2019), 351–369.

[26] S. Markov, A. Iliev, A. Rahnev, N. Kyurkchiev, A note on the n-stage growth model. Overview, *Biomath Communications*, **5**, No. 2 (2018), 79–100.

[27] E. Angelova, A. Golev, T. Terzieva, O. Rahneva, A study on a hyper-power-logistic model. Some applications, *Neural, Parallel, and Scientific Computations*, **27**, No. 1 (2019), 45–57.

[28] N. Kyurkchiev, A. Iliev, A. Rahnev, On a special choice of nutrient supply for cell growth in a continuous bioreactor. Some modeling and approximation aspects, *Dynamic Systems and Applications*, **28**, No. 3 (2019), 587–606.

[29] N. Kyurkchiev, A. Iliev, *Extension of Gompertz-type equation in modern science. 240 Anniversary of the birth of B. Gompertz*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-90569-0.

[30] N. Kyurkchiev, A. Iliev, A. Rahnev, T. Terzieva, Properties of a power Topp–Leone G–family with baseline Gompertz cumulative distribution function, (2020) (in print)

[31] N. Kyurkchiev, Comments on the Yun's algebraic activation function. Some extensions in the trigonometric case, *Dynamic Systems and Applications*, **28**, No. 3 (2019), 533–543.

[32] N. Kyurkchiev, G. Nikolov, Comments on some new classes of sigmoidal and activation functions. Applications, *Dynamic Systems and Applications*, **28**, No. 4 (2019), 789–808.

[33] N. Kyurkchiev, S. Markov, On a logistic differential model. Some applications, *Biomath Communications*, **6**, No. 1 (2019), 34–50.

[34] N. Kyurkchiev, A. Iliev, A. Rahnev, A special choice of nutrient supply for cell growth in logistic differential model. Some applications, *Conference Proceedings of AIP*, (2019)

[35] N. Kyurkchiev, A. Iliev, A. Golev, A. Rahnev, On a Special Choice of Nutrient Supply with Marshall-Olkin Correction. Some Applications, *Communications in Applied Analysis*, **23**, No. 3 (2019), 401–419.