Cell Proliferation Kinetics and Branching Stochastic Processes

Ollivier Hyrien[1], Kosto V. Mitov[2], Nikolay M. Yanev[3]
[1] University of Rochester
Ollivier_Hyrien@URMC.Rochester.edu
[2] Aviation Faculty - NMU
kmitov@yahoo.com
[3] Inst. of Mathematics and Informatics
yanev@math.bas.bg

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main purpose of this work is to present some new ideas and results obtained in modeling of cell proliferation kinetics. Recent advances in experimental techniques of flow cytometry have made it possible to collect a wealth of information about the status of individual cells isolated from dissociated tissues. When one is interested in modeling tissue development starting from the earliest embryonic stages it is reasonable to begin with 0 cells because these cells appear only in the course of embryogenesis. New type cells (immigrants) of age zero arrive in the population of cells in accordance with a non-homogeneous Poisson process with arrival rate $r(t)$. Upon arrival, these immigrants are assumed to be of age zero. Upon completion of its lifespan, every cell either divides into two new cells, or it goes out of the process of proliferation (differentiation or death). These two events occur with probability p and $q = 1 - p$, respectively. The time to division or differentiation of any cell is described by a non-negative random variable τ with c.d.f. $G(x) = \mathbf{P}\{\eta \leq x\}$. Cells are assumed to evolve independently of each other. Motivated by the above example, we investigate properties of a class of Markov branching processes with non-homogeneous Poisson immigration. We consider a more general process than the one presented above so that the scope of our work does not remain limited to the study of oligodendrocyte generation. Limiting distributions are obtained in the supercritical case and among them an analogue of the LLN and of the CLT.