Turing Instability of a Chemical System with Reactants Binding to a Substrate

Karolína Korvasová, Václav Klika
Czech Technical University in Prague
karolina.korvasova@fjfi.cvut.cz, klika@it.cas.cz

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Relevance of Turing mechanism for biology has been questioned due to not allowing pattern formation when the diffusion constants of the two reacting chemicals are identical. The idea that binding of the activator to a substrate may effectively reduce its diffusion rate and thus destabilize a system that would otherwise be stable was formulated in an article by Lengyel and Epstein [2], where the authors reduce the original system of three linear partial differential equations to a two-dimensional reaction-diffusion system that they analyse. We question relevance of this analysis due to lack of connection between the original and the reduced model and suggest that analysing the reduced model actually does not yield any possibilities beyond the standard setting. Nevertheless, our analysis of the three dimensional system shows that one can indeed relax the standard conditions on diffusion constants that are necessary for Turing instability, in particular allow identical diffusion coefficients. Another question that we raise is whether one can relax the condition on the effective diffusion rates in the three- or four-dimensional model. This idea is supported by a previous result of Klika, Baker, Headon and Gaffney [1] that one can significantly reduce the necessary conditions for Turing instability by adding more reactants into the system. Furthermore, the approach of studying the full four-dimensional system allows relaxing the kinetic contraints, for example by permitting two activators to generate a pattern.

References

[1] V. Klika, R. E. Baker, D. Headon, E. A. Gaffney, *The Influence of Receptor-Mediated Interactions on Reaction-Diffusion Mechanisms of Cellular Self-organisation*, Bull Math Biol (2012), **74** 935–957.

[2] I. Lengyel, I. R. Epstein, *A Chemical Approach to Designing Turing Patterns in Reaction-Diffusion Systems*, Proc. Natl. Acad. Sci. USA (1992), **89** 3977–3979.