The Impact of Allee Effect on Infectious Disease Dynamics

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It is well known that the Allee effect contributes to developing complicated infectious disease dynamics like multistability, oscillations and disease induced extinction of the host population. Here we characterize and quantify its epidemiological impact within a relatively simple SI model excluding vertical transmission. In such setting, the vital dynamics (births and deaths), which are responsible for the Allee effect, affect significantly the course of the disease. In the presented model the birth rate and the death rate are both modeled as quadratic polynomials. This approach provides ample opportunity for taking into account the major contributors to the Allee effect (mating opportunities, cooperative feeding, joint defence, reduced exposure to predators, cooperation in raising the young) and is an extension of the model presented in [1]. We determine two essential threshold values λ_0 and λ_1 of the infectiousness/transmissibility λ of the disease. For $\lambda \leq \lambda_0$ the disease free state is stable and attractive and the model exhibits the same bistability as the one of the the disease free state. Under certain conditions the Allee effect causes epidemic like behavior as the disease runs its course to extinction. When $\lambda > \lambda_1$ the disease free state is also stable and attractive but the origin is the only stable equilibrium. Hence introducing the disease at any population size leads to extinction. For λ between the two thresholds the eventual outcome of introducing the disease, namely an endemic state or a population extinction, depends on the population size. Some conditions distinguishing between the two outcomes have been derived.

[1] F. M. Hilker, M.Langlias, H. Malchow, *The Allee effect and infectious diseases: Extinction, multiplicity, and the (dis-)appearance of oscillations*, American Naturalist **173** (2009) 72–88.