

# Backward Bifurcation for Pulse Vaccination

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In this talk we investigate an SIVS epidemic model with imperfect vaccine, thus vaccinated individuals can also contract the infection. We consider pulse vaccination, that means we vaccinate a large fraction of population at fixed time intervals.

It is known that in some vaccination models, backward bifurcation occurs and multiple subthreshold endemic equilibria exist, thus the behaviour of solution depends on the initial value as well [1]. We know that pulse vaccination can be more effective than constant vaccination, thus it is an interesting question to study whether backward bifurcation can arise in a pulse vaccination model.

We proved that backward bifurcation can occur in pulse vaccination. First we found the disease-free periodic solution, which is locally asymptotically stable in the whole phase space if the control reproduction number less than one; if the control reproduction number greater than one, then the infection is strongly uniformly persistent in the population.

We performed the complete bifurcation analysis of a fixed point equation, where the most important tool was the Lyapunov-Schmidt method and we obtained a sufficient and necessary condition for the existence of subcritical bifurcation.

## References

- [1] F. Brauer, *Backward bifurcation in simple vaccination models*, J. Math. Anal. Appl. **298** 418–431 .
- [2] Zs. Vizi, G. Röst *Backward bifurcation for pulse vaccination* (submitted)