Pattern Formation in a Reaction-Diffusion System with a Singularity

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Systems of reaction-diffusion equations are widely used to model morphogenesis (organisation of forms and patterns in living organisms) in different biological contexts. A particular application describes the formation of animal coat patterns and distribution of structures on the epidermis. The patterns of interest are either an arrangement of local maxima or a stripe-like distribution of chemical concentrations over the domain of interest that persist over time. In mathematical terms, a pattern is a spatially inhomogeneous solution of the reaction-diffusion system which is asymptotically stable. The common feature of systems displaying pattern formation is the presence of a steady state that is asymptotically stable to spatially-homogeneous perturbations but asymptotically unstable to spatially-inhomogeneous perturbations [1]. We study whether it is possible to relax this assumption with an analysis of a reaction-diffusion system with a singularity that is a slight modification of a model for hair follicle spacing [2]. We prove existence of global solutions for the reaction-diffusion system and examine stability of spatially inhomogeneous stationary solutions.

References

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