Contributions to Constructive Treatment of SIS-Type Models II: Nonstandard Volterra Difference Equation

Jean M.-S. Lubuma, Yibeltal A. Terefe
Department of Mathematics and Applied Mathematics
University of Pretoria, Pretoria 0002, South Africa
jean.lubuma@up.ac.za; yibeltal.terefe@up.ac.za

Keywords: Diffusion, Dynamics Preserving Nonstandard Finite Difference Scheme, Forward/backward bifurcation, Volterra Difference Equation.

This talk given by the first author is the second in a series of two. The first part will be presented by the other author, though the abstract is reproduced below for the convenience. We deal with various reliable nonstandard finite difference (NSFD) schemes for the SIS epidemiological model in different settings.

In the first part, we construct and investigate the reliability of various NSFD schemes for diffusion and space-independent models. We consider two new NSFD schemes for the SIS-ODE model, which faithfully replicate the property of the continuous model of having the value $\mathcal{R}_0 = 1$ of the basic reproduction parameter as a forward bifurcation: the disease-free equilibrium (DFE) is globally asymptotically stable (GAS) when $\mathcal{R}_0 < 1$; it is unstable when $\mathcal{R}_0 > 1$ and there appears a unique locally asymptotically stable (LAS) endemic equilibrium (EE) in this case. The schemes are further used to derive NSFD schemes that are dynamically consistent with the positivity and boundedness properties of the SIS-diffusion model.

In the second part, the contact rate is a function of the infective population and we incorporate a distributed infective period. The resulting SIS-model is a Volterra integral equation of the second kind. The qualitative analysis is now based on two threshold parameters $\mathcal{R}_0^c \leq 1 \leq \mathcal{R}_0^m$ and the system can undergo the backward bifurcation as follows. The DFE is the only equilibrium and it is GAS when $\mathcal{R}_0 < \mathcal{R}_0^c$; there exists only one EE, which is LAS when $\mathcal{R}_0 > \mathcal{R}_0^m$ with the DFE being unstable when $\mathcal{R}_0 > 1$; for $\mathcal{R}_0^c < \mathcal{R}_0 < 1$, the DFE is LAS and co-exists with at least one LAS endemic equilibrium. We design a NSFD scheme that preserves positivity and boundedness of the solution as well as the above-stated stability properties of equilibria. Numerical simulations that support the theory are provided.