Singular Perturbations and Biomathematics

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In the first part of the talk examples of singular perturbation problems in biomathematics are presented. Then some mathematical properties of singular perturbation problems are introduced and analytical techniques for their solution are discussed. In most cases it is necessary to use numerical methods. It is demonstrated that standard numerical methods for singularly perturbed problems are not adequate for computing reliable approximations to their exact solution. Specially designed numerical methods comprising standard numerical operators on Shishkin meshes are introduced. It is demonstrated, by numerical examples, that these methods are adequate for solving some typical singularly perturbed problems. In conclusion a singularly perturbed system of semilinear delay differential equations, arising from a problem in biomathematics, is formulated. It is then solved using a parameter uniform numerical method, whose solutions converge robustly to the exact solution of the problem.