Global Stability of a Two Patch Cholera Model with Fast and Slow Transmission

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Global stability of epidemic patch models is always a challenging mathematical issue. For a waterborne disease such as Cholera, we extend a model originated from [1] in three ways: we consider a two patches environment, add disease induced mortality and migration of humans between the patches. We give the conditions under which the model exhibit four different equilibria and show that their existence is based on the local basic reproduction numbers and the type-reproduction numbers of the two patches. Using technical Lyapunov functions that combine quadractic, Volterra-type and linear functions, we prove the global asymptotic stability of both the disease free equilibrium and the two boundrary equilibria in the positive orthant. As for the endemic equilibrium, we use a Volterra-type Lyapunov function to prove its global stability in the positive orthant under biological reasonable conditions on model parameters.

References

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