An Approach to the Interval Arithmetic from a Probabilistic Point of View

Petar Konovski
Birkbeck, University of London
petar@dcs.bbk.ac.uk

The interval arithmetic is a popular tool which the scientists apply to deal with inexact and uncertain sets of data as well as with the inevitable rounding errors during the scientific computations.

Separately and almost without exchanging ideas with the interval arithmetic, a different approach to the same scientific problems emerged: The stochastic arithmetic.

Comparing its popularity with the popularity of the interval arithmetic, it can be qualified as marginal, though that approach deserves much more attention.

In the cases where in the interval arithmetic the data are represented with intervals, in the stochastic arithmetic they are represented with distributions, similar to the Gauss distributions (named there "stochastic numbers").

The idea which promises to conform those two approaches is to consider the initial intervals participating in the interval arithmetic calculation as continuous uniform distributions, i.e., simply a particular case of random variables. In this situation, after any step of the calculation, instead of turning the result back in intervals (continuous uniform distributions), and losing information, we can continue the calculations with the newly obtained random variables. Several examples are given which illustrate the benefits of this approach.

The practical implementation of that new approach is borrowed from even more exotic branch of the numerical analysis: The idea for numerical computation with functions instead of numbers. For now, it is represented by the project Chebfun (developed by its team from the Oxford University Mathematical Institute). Their main result is the numerical package Chebfun.

The basic idea of the project Chebfun is that an arbitrary numerical (and preferably continuous) function is approximated by a Chebyshev polynomial of sufficiently high degree and after that the polynomial represent the function in any further calculations.