

# Hopf Bifurcations of a Ratio-Dependent Predator-Prey Model Involving Two Discrete Maturation Time Delays

Esra Karaoglu, Huseyin Merdan  
Department of Mathematics, TOBB University of Economics and  
Technology, Ankara, Turkey  
ekaraoglu@etu.edu.tr, merdan@etu.edu.tr

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In this paper we give a detailed Hopf bifurcation analysis of a ratio-dependent predator-prey system involving two different discrete delays. By analyzing the characteristic equation associated with the model, its linear stability is investigated. Choosing delay terms as bifurcation parameters the existence of Hopf bifurcations is demonstrated. Stability of the bifurcating periodic solutions is determined by using the center manifold theorem and the normal form theory introduced by Hassard et al. Furthermore, some of the bifurcation properties including direction, stability and period are given. Finally, theoretical results are supported by some numerical simulations.

## References

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