A Validated Model of the Pro- and Anti-inflammatory Cytokine Balancing Act in Articular Cartilage Lesion Formation under Mechanical Strain

Georgi Kapitanov¹ Xiayi Wang², Bruce P.Ayati^{1,2,3}, Marc J.Brouillette^{3,4}, James A.Martin^{3,4} ¹ Department of Mathematics, University of Iowa ² Program in Applied Mathematical & Computational Sciences, University of Iowa ³Department of Orthopaedics & Rehabilitation, University of Iowa ⁴Department of Biomedical Engineering, University of Iowa georgi-kapitanov@uiowa.edu xiayi-wang@uiowa.edu bruce-ayati@uiowa.edu james-martin@uiowa.edu

Keywords: articular cartilage, structured model, lesion formation and abatement, EPO, IL-6

Joint (articular) cartilage is designed to sustain certain level of impact. However, when a large enough force is applied, the cells in the cartilage release pro- and anti-inflammatory chemicals that can affect the cell population, the chemical feedback loop, as well as the cartilage itself, causing lesions. Modeling all these processes requires several components: cellular, chemical, and mechanical. The cellular component deals with the population densities of different cells and their interactions and transitions from one type into another. These transitions are induced by, and in turn affect, the chemicals which are released as a result of the impact. The first two components have already been modeled in published papers, using a nonlinear system of age-structured, parabolic PDEs with a radial spacial dimension. The next step is modeling the mechanics of the impact as more than a constant force term, but as a time-dependent effect on the physical and chemical processes in the cartilage. This requires an additional spacial dimension for the depth of the cartilage. Increasing the dimensions, besides adjusting the original equations accordingly, requires the addition of a system of hyperbolic PDEs, to model the effect of the impact using linear elasticity. The current model develops and solves the system numerically, and is validated by experimental results.