

# Numerical Framework for Pattern-Forming Models on Evolving-in-Time Surfaces

Ramzan Ali, Andriy Sokolov, Robert Strehl and Stefan Turek  
Institut für Angewandte Mathematik, TU Dortmund  
Vogelpothsweg 87  
44227 Dortmund, Germany

ramzan.ali@math.tu-dortmund.de, asokolow@math.tu-dortmund.de,  
robert.strehl@math.tu-dortmund.de, stefan.turek@math.tu-dortmund.de

*Keywords: Chemotaxis, Turing Instability, Pattern Formation, Level Set, FEM.*

In this contribution we describe a numerical framework for a system of coupled reaction-diffusion equations on an evolving-in-time hypersurface  $\Gamma$ . Numerical tests are employed for Turing-type instability on stationary and evolving surfaces. The proposed framework combines the level set methodology for the implicit description of the time dependent  $\Gamma$  [1], the Eulerian finite element formulation for the numerical treatment of partial differential equations [2] and the flux-corrected transport scheme [3] for the numerical stabilization of arising advective, resp., convective terms. Major advantages of this scheme are that it avoids numerical calculation of curvature, allows coupling of surface-defined partial differential equations with domain-defined partial differential equations through the level set bulk and preserves the positivity of the solution through the algebraic flux correction. Corresponding numerical tests demonstrate the ability of the scheme to deliver highly accurate solutions with a reasonably good convergence behavior.

## References

- [1] A. Sokolov, R. Ali, S. Turek, *An AFC-stabilized implicit finite element method for partial differential equations on evolving-in-time surfaces*, J. Comput. Appl. Math., 2015 (accepted) doi : 10.1016/j.cam.2015.03.002.
- [2] C.M. Elliott, B. Stinner, C. Venkataraman, *Modelling cell motility and chemotaxis with evolving surface finite elements*, Journal of Royal Society Interface, 2012 **9(67)** 3027–3044.
- [3] R. Strehl, A. Sokolov, D. Kuzmin, S. Turek, *A flux-corrected finite element method for chemotaxis problems*, Comput. Methods Appl. Math., 2010 **10(2)** 219–232.