Peristaltic Transport of Viscoelastic Bio-fluids with Fractional Derivative Models

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Peristaltic transport occurs in many biological and biomechanical systems. In this work, a mathematical study of the peristaltic flow of a viscoelastic fluid through a non-uniform channel is presented under the assumptions of long wavelength and low Reynolds number. The viscoelastic properties of the fluid are modeled by using the fractional Burgers' constitutive equation, containing four fractional Riemann-Liouville derivatives of orders $\alpha, 2\alpha, \beta$ and 2β, where $0 < \alpha \leq \beta < 1$. This model covers the fractional Oldroyd-B, Maxwell and second-grade fluid as particular cases. Models of peristaltic transport under such assumptions are given in a series of recent papers by Tripathi *et al.*, see e.g. [1] and the references therein.

According to the above models, the pressure gradient can be obtained as a solution of a differential equation of fractional order. Our main contribution is a detailed study of this equation. Applying Laplace transform, its analytical solution is represented in terms of Mittag-Leffler functions. To compute the numerical solution, some numerical techniques are presented and compared. The results of several of numerical examples are given and the effects of the different parameters on the pressure difference across one wavelength are discussed.

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References

[1] D. Tripathi, O. Anwar Bég, *Peristaltic propulsion of generalized Burgers fluids through a non-uniform porous medium: A study of chyme dynamics through the diseased intestine*, Math. Biosci. **248** (2014) 67–77.