Stability and Instability of Improved Heimburg–Jackson Model to Nerve Pulse Propagation

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There are a number of mathematical models to nerve pulse propagation in biomembranes, as Hodgkin–Huxley, FitzHugh–Nagumo and Heimburg–Jackson models, see, e.g., [1,2]. However, these models do not describe adequately all observed phenomena. Recently in [2], generalized Boussinesq equation with quadratic–cubic nonlinearity, i.e.,

$$u_{tt} - u_{xx} + h_1 u_{xxxx} - h_2 u_{ttxx} + (au^2 + bu^3)_{xx} = 0 \tag{1}$$

is proposed as an improvement of the well–known Heimburg–Jackson model ($h_2 = 0$).

In this study we prove analytically the orbital stability and instability of solitary waves to the improved Heimburg–Jackson model (1). The results depend on the relationship between the parameters h_1, h_2, a, b. For the set of data, obtained experimentally, our theoretical results are in full agreement with the numerical simulations, presented in [3].

References

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